PROCEEDINGS

American Society of Civil Engineers

JANUARY

1947

AMERICAN SOCIETY OF CIVIL ENGINEERS FOUNDED 1852

VOLUME 73

NO. 1

PROCEEDINGS

OF THE

AMERICAN SOCIETY OF CIVIL ENGINEERS

VOL. 73 JANUARY, 1947 No. 1

TECHNICAL PAPERS

AND

DISCUSSIONS

Published monthly, except July and August, at Prince and Lemon Streets, Lancaster, Pa., by the American Society of Civil Engineers. Editorial and General Offices at 33 West Thirty-Ninth Street, New York 18, N. Y. Reprints from this publication may be made on condition that the full title of Paper, name of Author, page reference, and date of publication by the Society are given.

Entered as Second-Class Matter, September 23, 1937, at the Post Office at Lancaster, Pa., under the Act of March 3, 1879. Acceptance for mailing at special rate of postage provided for in Section 1103, Act of October 3, 1917, authorized on July 5, 1918.

Subscription (if entered before January 1) $8.00 per annum Price $1.00 per copy

	Published	Discussion closes
Symposium: Cavitation in Hydraulic Structures	Sept., 1945	
Discussion in Dec., 1945, Feb., Mar., Apr., May, June, Oct., 1946		Closed*
Freudenthal, Alfred M. The Safety of Structures	Oct., 1945	
Discussion in Jan., Feb., Apr., June, 1946		Closed*
Symposium: Uplift Pressure in and Beneath Dams	Dec., 1945	
Discussion in June, Sept., 1946		Closed*
Senour, Charles. New Project for Stabilizing and Deepening Lower Mississippi River	Feb., 1946	
Discussion in Sept., Nov., 1946		Closed*
Ripperger, E. A., and Davids, N. Critical Stresses in a Circular Ring	Feb., 1946	
Discussion in Oct., Nov., 1946		Closed*
Forbes, Hyde. Landslide Investigation and Correction	Feb., 1946	
Discussion in May, Sept., Nov., Dec., 1946		Closed*
Winter, George. Strength of Thin Steel Compression Flanges	Feb., 1946	
Discussion in June, Oct., Dec., 1946, Jan., 1947		Closed*
Lothers, J. E. Torsion in Steel Spandrel Girders	Mar., 1946	
Discussion in June, Oct., Nov., 1946		Closed*
Wood, Charles P. Factors Controlling the Location of Various Types of Industry	Mar., 1946	
Discussion in Nov., 1946		Closed*
Barnett, Joseph. Express Highway Planning in Metropolitan Areas	Mar., 1946	
Discussion in May, Sept., Nov., 1946, Jan., 1947		Closed*
Howson, Louis R. Future Costs and Their Effects on Engineering Budgets	Mar., 1946	
Discussion in Jan., 1947		Closed*
Sette, F. J. The Planning of Aerial Photographic Projects	Mar., 1946	
Discussion in Sept., Oct., 1946		Closed*
Report: Engineering Education: Progress Reports of the Society's Professional Committee on Engineering Education for 1944 and 1945	Mar., 1946	
Discussion in June, Sept., Oct., 1946, Jan., 1947		Closed
Dunham, John W. Design Live Loads in Buildings	Apr., 1946	
Discussion in Oct., 1946		Feb. 1, 1947
Peterson, F. G. Eric. Effect of Stress Distribution on Yield Points	Apr., 1946	
Discussion in Sept., Nov., Dec., 1946		Feb. 1, 1947
Baker, Donald M. Some Thoughts on Engineering Education	Apr., 1946	
Discussion in Sept., Oct., Dec., 1946		Feb. 1, 1947
Report: Topographic Surveys: Progress Report of the Committee of the Surveying and Mapping Division on Topographic Surveys	Apr., 1946	
Discussion in Oct., Nov., Dec., 1946		Feb. 1, 1947
Rose, Edwin. Thrust Exerted by Expanding Ice Sheet	May, 1946	
Discussion in Dec., 1946		Feb. 1, 1947
Dell, George H. Moment-Stiffness Relations in Continuous Frames with Prismatic Members	May, 1946	
Discussion in Oct., 1946		Feb. 1, 1947
Bermel, Karl J., and Sanks, Robert L. Model Study of Brown Canyon Debris Barrier	May, 1946	
Discussion in Nov., Dec., 1946		Feb. 1, 1947
Jacob, C. E. Effective Radius of Drawdown Test to Determine Artesian Well	May, 1946	
Discussion in Nov., Dec., 1946		Feb. 1, 1947
Report: Opportunities for Sanitary Engineers in the Postwar Period: Progress Report of the Committee on Advancement of Sanitary Engineering of the Sanitary Engineering Division	May, 1946	
Moore, R. L. Observations on the Behavior of Aluminum Alloy Test Girders	June, 1946	Feb. 1, 1947
Hansen, Howard J. Design of Plywood I-Beams	June, 1946	
Discussion in Nov., 1946, Jan., 1947		Feb. 1, 1947
Tommerup, Carl C. H. Rigid-Frame Structures Subject to Nonuniform Thermal Action	June, 1946	
Discussion in Nov., 1946, Jan., 1947		Feb. 1, 1947
Middlebrooks, T. A., and Jervis, William H. Relief Wells for Dams and Levees	June, 1946	
Discussion in Oct., Dec., 1946, Jan., 1947		Feb. 1, 1947
Report: Advances in Sewage Treatment and Present Status of the Art: Third Progress Report of the Committee of the Sanitary Engineering Division on Sewerage and Sewage Treatment	June, 1946	
Stevens, J. W. Truck Speed and Time Loss on Grades	Sept., 1946	
Discussion in Dec., 1946		Mar. 1, 1947
Underwood, P. H. Space Resection Problems in Photogrammetry	Sept., 1946	
Discussion in Dec., 1946, Jan., 1947		Mar. 1, 1947
McGuinness, Charles L. Recharge and Depletion of Ground-Water Supplies	Sept., 1946	Mar. 1, 1947
de Vries, Karl. Strength of Beams as Determined by Lateral Buckling	Sept., 1946	
Discussion in Dec., 1946		Mar. 1, 1947
Ward, C. N., and Hunt, Henry J. Correction of Tailwater Erosion at Prairie Du Sac Dam	Oct., 1946	Mar. 1, 1947
Benscoter, Stanley U. Matrix Analysis of Continuous Beams	Oct., 1946	Mar. 1, 1947
Machis, Alfred. Experimental Observations on Grouting Sands and Gravels	Nov., 1946	Apr. 1, 1947
Symposium: Cleaning and Grouting of Limestone Foundations, Tennessee Valley Authority	Dec., 1946	May 1, 1947

NOTE.—The closing dates herein published are final except when names of prospective discussers are registered for special extension of time.

* Publication of closing discussion pending.

Eng. Lib
direct

Vol. 73 PROCEEDINGS No. 1

CONTENTS FOR JANUARY, 1947

PAPERS

DISCUSSIONS

CONTENTS FOR JANUARY, 1947 (*Continued*)

A list of "Current Papers and Discussions" may be found on the page preceding the table of contents

HINTS TO AUTHORS

Those who are planning papers for submission to PROCEEDINGS *will expedite Committee action measurably by first studying the standard instructions as to style, content, and format. For reprints, address the Manager, Technical Publications, 33 W. 39th St., New York 18, N. Y.*

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

MISSISSIPPI RIVER CUTOFFS

By Gerard H. Matthes,[1] Hon. M. ASCE

Synopsis

Natural and artificial cutoffs on the Lower Mississippi River in the 50-mile stretch north of Vicksburg, Miss., are described with respect to their effect on river shortening and flood-stage lowering. Historical and physical facts are given which explain why, during the period from 1884 to 1932, natural cutoffs were dreaded and engineering measures were taken to prevent their occurrence, thus permitting the river to become considerably lengthened. The paper describes the unforeseen circumstances which caused a cutoff at Yucatan Bend to occur unexpectedly during the low-water season of 1929; and it relates how the satisfactory manner in which the cutoff developed unassisted gave impetus to the launching of a program of river shortening by artificial cutoffs, the first of which, across Diamond Point, was opened on January 8, 1933. The technique adopted avoided making cutoffs across narrow necks, except where conditions made this necessary, and also avoided the pitfalls of European practice. Since that date fourteen additional cutoffs have been constructed, making a total of sixteen. In connection with other forms of channel rectification and dredging operations, these cutoffs have shortened the Lower Mississippi a total of 170 miles between Memphis, Tenn., and Baton Rouge, La., in an original 680-mile river length, or 25%. The appreciable lowering of flood stages which resulted from the cutoffs has saved vast outlays in levee construction. The cutoff data presented are from the records of the Mississippi River Commission.

Historical

In early times, the Lower Mississippi River, operating without restraint, made cutoffs at the rate of from about thirteen to fifteen in the course of a century. These cutoffs usually occurred during very high stages when water

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **June 1, 1947.**

[1] New York, N. Y.; formerly Head Engr., Mississippi River Comm., and Director, U. S. Waterways Experiment Station, Vicksburg, Miss.

scoured out channels across the narrow necks of overdeveloped bends. Individual shortenings of the river channel so produced ranged between 6 miles and as many as 21 miles. An assumption of fifteen cutoffs and an average net shortening of 12 miles per cutoff would represent a total shortening of 180 miles in a century, or 1.8 miles a year. As applied to the 840-mile stretch of river between Cairo, Ill., and Baton Rouge (cutoffs have not occurred below the latter point, see Fig. 1) 1.8 miles per year, or any value like it, seems totally inadequate to compensate for the habitual lengthening of the river, which is known to be more nearly on the order of 4 or 5 miles per year. Thus, other shortening processes must account for the difference. The most obvious of these is the channel shortening effected by chute developments across point bars. These developments result from the gradual enlargement of shallow swales (which began during high stages) into channels deep enough to flow water even at low stages and, in time, the swales become main-river channels. Chute developments have been numerous; but, because they were mostly slow processes and therefore not as spectacular as natural cutoffs, they have attracted little attention. Appreciable river shortenings have resulted from them, however. In most instances, the gradual enlargement of chutes has been conducive to producing better channels than have resulted from quick-acting natural cutoffs. The latter, because of their disruptive nature, as a rule were followed by rapid and drastic channel changes often involving major property losses and inconvenience to navigation.

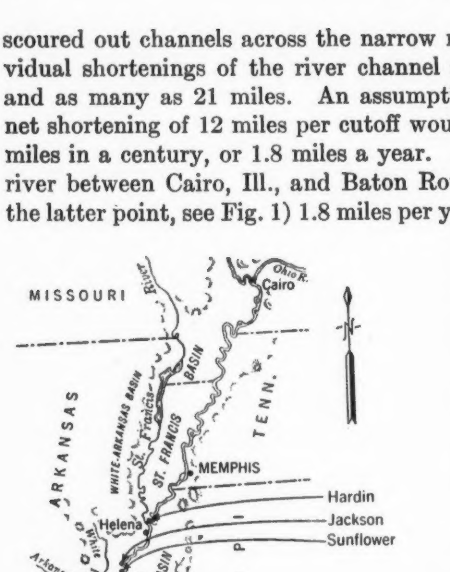


FIG. 1.—LOWER MISSISSIPPI RIVER

In addition to cutoffs and chute developments, river shortenings have resulted from bend flattening. This phenomenon has been of minor importance but is of interest in that it dispels the popular belief that river bends always tend to assume greater, and eventually excessive, curvature.

One cannot but marvel at the consistency with which these natural processes have operated to maintain the over-all length in the Cairo-to-Baton Rouge stretch about the same through the long course of time: 884 miles according to

maps compiled from 1820–1825 survey data; 833 miles according to the first Mississippi River Commission survey of 1881–1882; 842 miles according to the 1916 survey; and 846 miles according to the 1929–1930 survey.

When, on January 8, 1933, Gen. Harley B. Ferguson, M. ASCE, then President of the Mississippi River Commission, opened the first pilot cut across Diamond Point (Fig. 1), the erstwhile policy of "no cutoffs at any price" maintained by the Mississippi River Commission over a period of 48 years went into the discard. The events which originally caused the Commission in 1884 to adopt its "no cutoff" stand briefly were these: During the 1870's three

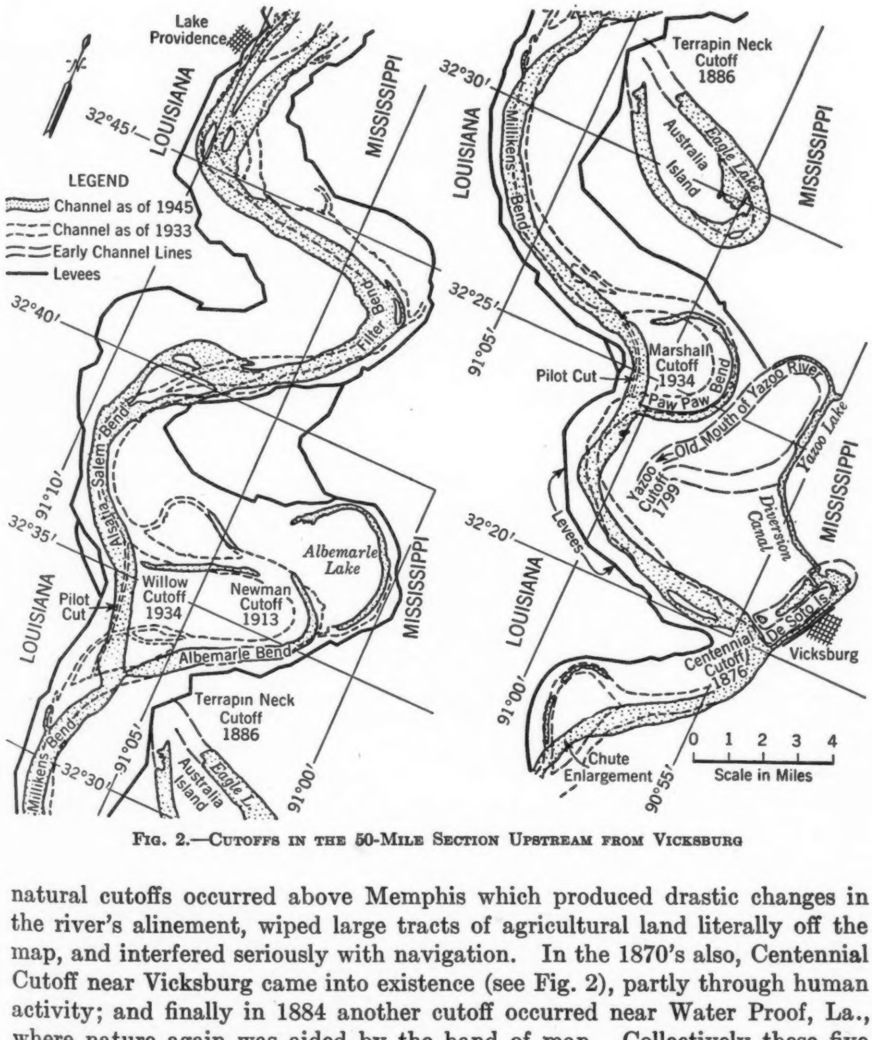


FIG. 2.—CUTOFFS IN THE 50-MILE SECTION UPSTREAM FROM VICKSBURG

natural cutoffs occurred above Memphis which produced drastic changes in the river's alinement, wiped large tracts of agricultural land literally off the map, and interfered seriously with navigation. In the 1870's also, Centennial Cutoff near Vicksburg came into existence (see Fig. 2), partly through human activity; and finally in 1884 another cutoff occurred near Water Proof, La., where nature again was aided by the hand of man. Collectively these five

cutoffs shortened the river an aggregate of 50 miles within one decade. The engineers of the Mississippi River Commission were powerless to direct or control the river's rampaging.

The Commission's policy to permit no further natural cutoffs to occur found justification in the fact that the Commission in those days had neither the funds nor the dredging equipment at its command with which to make artificial cutoffs or to control their development. Moreover, natural cutoffs across narrow necks of land as a rule produced only temporary lowerings of flood stages and induced successions of channel changes extending upstream as well as downstream. To prevent impending cutoffs from taking place, the Commission caused dikes to be built on Ashbrook Neck and Leland Neck near Greenville, Miss., and on the neck of Cowpen Point near Natchez, Miss., later isolated by Giles Cutoff.

Proposals to improve the Lower Mississippi River by artificial cutoffs had been made from time to time dating as far back as 1850; in 1882 the late J. B. Johnson, M. ASCE, assistant engineer for the Commission, submitted such a recommendation. After the 1927 flood, the desire for making artificial cutoffs to lower the flood stages gained new impetus. The late John F. Coleman, Past-President and Hon. M. ASCE, strongly supported a well-coordinated cutoff program, but expressed the need for devising safe methods for executing such works. W. E. Elam, M. ASCE, discussed a paper [2] in which he showed the benefits that could be derived by cutting off the Greenville Bends—but always the question of how to execute and control such operations in as large a river as the Mississippi remained unanswered. Not until 1930 did General Ferguson, then Colonel, Corps of Engineers, and Division Engineer for the South Atlantic Coast, come forward with what appeared to be a sound plan of procedure. Instead of adopting the European technique of making cutoffs in the dry to full dimensions and then turning the river into them, he devised the pilot-cut plan. This plan permitted the slope adjustment incident to the river shortening to develop gradually over a period of years, and avoided creating high velocities and raising flood stages in the river downstream from the cutoff—two evils which had invariably resulted from the European method.

In adopting his plan General Ferguson was guided by the development of Yucatan Cutoff—which cut off Hard Times Bend unexpectedly in September, 1929. This cutoff occurred at a point about 40 miles downstream from Vicksburg (see Fig. 1) when the Mississippi River, through the caving of its banks, broke into its tributary, the Big Black River, at a point 2.5 miles above the latter's mouth. As the mouth was about 12 miles downstream from the break through by way of Hard Times Bend, the entire fall around this bend became concentrated in the 2.5-mile Big Black channel. Although the Commission's engineers had endeavored to close the break with willow mats, the flow gained so much headway that it could not be stopped. There resulted what subsequent events have shown to be the model cutoff on the Lower Mississippi. After two flood seasons the crooked little valley of Big Black River had become transformed into a wide and nearly straight cutoff channel fit for navigation. By

[2] *Transactions*, ASCE, Vol. 93, 1929, p. 937.

April, 1932, it was carrying 60% of the river's flow; it was more than 100 ft deep in places; and this great depth gave so ample a cross section that the rate of fall was almost the same as in the river above and below the cutoff. Consequently, velocities were moderate and offered no obstacles to navigation. Concurrently with this development, deterioration of the old channel around Hard Times Bend had progressed, through deposition of sand, until, by November 22, 1933, during a low-river stage, the entire flow was passing through Yucatan Cutoff and the writer was able on that date to walk dry-shod across the severed bend channel at its upper end.

The Yucatan incident, aside from being extraordinary as a natural avulsion (occurring during low-river, instead of high-river, stages) set a new precedent in cutoff history. It proved that a narrow channel a mile or two in length was superior, as a cutoff route, to a short cut across a narrow neck of land. Yucatan Cutoff developed without any assistance from the engineers, and has not upset the river either upstream or downstream in a detrimental way. Such channel changes as did result were not appreciably different from those ordinarily witnessed in the river's meandering.

The program of cutoffs inaugurated in 1933 has followed this precedent wherever conditions permitted it. It was aimed at shortening the river without, however, straightening it. Mild curvature was held to be essential for preserving a deep navigable channel as well as for effecting such channel stabilization as might be consistent with the meandering nature of an alluvial river. The main objective, however, was the lowering of flood stages; and, in this respect, the cutoff program has made an extraordinary contribution to the control of floods in the Lower Mississippi River. More than that, it was so planned as to make it incumbent on the waters of the Mississippi River to perform the major part of the work of excavating new channels and of filling abandoned ones. The program has effected stupendous savings in the construction costs of the levee system.

Natural Cutoffs

Six cutoffs that have occurred in the 50-mile reach upstream from Vicksburg afford a unique opportunity to analyze this subject (see Fig. 2)—Centennial (1876), Yazoo (1799), Terrapin Neck (1886), Newman (1913), Marshall (1934), and Willow (1934). The first four were cut by nature whereas the remaining two were created by man. For the most part, the paper is concerned with the latter two—Marshall and Willow cutoffs.

Centennial Cutoff.—In 1876, at Vicksburg (see Fig. 2), the Centennial Cutoff moved the river away from Vicksburg and created De Soto Island, which still remains a part of the State of Louisiana. Although this is a natural cutoff, the process was aided artificially.

Yazoo Cutoff.—Only a small part of the original Yazoo Bend, which was cut off in 1799, remains as Yazoo Lake.

Terrapin Neck Cutoff.—This action, in 1886, eliminated 16 miles of the original river channel, creating Eagle Lake in the process. Australia Island, within the loop formed by the lake, remains as a part of the State of Louisiana.

Newman Cutoff.—This development was not a cutoff in the true sense, but a natural chute development caused by the enlargement of Albemarle Chute in 1913.

Marshall Cutoff

Marshall Cutoff extends across Marshall Point around which the river formerly flowed in an almost perfect semicircular bend known as Paw Paw Bend. This bend had become subject to chronic bank caving and in 1932 threatened to effect a junction with the abandoned channel of the Yazoo River. It became necessary either to revet 3 miles of Paw Paw Bend or to make a cutoff. The decision was in favor of the cutoff.

The method of construction consisted in first excavating the high ground with dragline machines and then finishing the pilot cut down to a depth of 29 ft below mean low water with cutter-head dredges. Four draglines began operations on October 19, 1933, and worked a total of 80 machine days. On December 17, 1933, the high ground yardage removed was 1,841,000 cu yd. This cut was 6,000 ft long. Between November 12, 1933, and February 20, 1934, a cutter-head dredge worked 101 days removing 2,950,000 cu yd to open a pilot cut across the huge sand bar at the lower end for a distance of 4,400 ft. The cost of excavating this part of the pilot cut instead of trusting the river to wash out a channel of its own through the sand bar has been amply justified by the excellent performance of the cutoff channel in enlarging itself in the exact place it was designed to occupy. During February and March, 1934, a dredge worked 20 days removing 551,000 cu yd. The width of the completed pilot cut was 250 ft on the bottom, which represented the full swing that the dredges could make at one operation. The side slopes were left as steep as the soil would stand.

When the pilot cut was opened on March 12, 1934, the over-all length was 13,600 ft. However, to compute the amount by which the river was being shortened, it was necessary to add to the actual length of the pilot cut the distance to the axis of the main river at each end. This computation gave the navigable length of the cutoff as 3.1 miles. Deducting this length from the 7.3 miles around Paw Paw Bend made the net shortening 4.2 miles. The fall in the cutoff, at time of opening, was 2.2 ft or about 0.7 ft per mile. The river was not high and the resulting rush of water was not damaging, but the bed and banks of the pilot cut began to erode immediately, causing much earth to be entrained. Rising river stages augmented the flow through the cut; and, on March 27, 15 days after the opening, 60,500 cu ft per sec, or 8% of the flow in the Mississippi River, was passing through the cutoff. By April 10 the flow had increased to 84,000 cu ft per sec, or 10% of the river discharge. When river stages fell, the percentage of flow in the cutoff decreased. On May 28, at a low-river stage, only 18,600 cu ft per sec, or 7% of the river discharge, was passing and this percentage prevailed throughout the low-water season. Table 1(*a*) is a condensed record of how the discharge capacity increased during the course of time as the cutoff channel enlarged itself from a pilot cut 250 ft wide to a sizable river. While this enlargement progressed, the river channel in Paw Paw Bend shoaled and contracted without any aid from man.

TABLE 1.—Performance Statistics, Artificial Cutoffs on the Mississippi River

(a) Marshall Cutoff

Date of observation (1)	Gage height[a] (ft) (2)	Discharge in Cutoff: Cu ft per sec (3)	Discharge in Cutoff: Percentage[b] (4)
1934:			
March 27	30.9	60,500	8
April 9	33.3	84,000	10
May 28	8.0	18,600	7
August 13	3.4	10,900	7
1935:			
February 9	33.8	133,000	15
May 22	40.2	269,000	25
July 5	45.8	404,000	31
September 20	14.2	179,000	49
October 23	1.9	86,000	61
December 16	14.1	190,000	50
1936:			
March 12	27.7	315,000	42
April 23	42.6	517,000	42
May 29	12.5	229,000	66
September 1	3.1	83,900	88
December 18	10.4	229,000	67
1937:			
January 26	41.1	608,000	50
February 17	55.1	940,000	47
July 21	10.6	322,000	99
August 10	6.2	247,000	100
During low water	....	....	100
1938:			
January 4	20.7	487,000	84
April 20	41.1	757,000	62
June 8	34.1	704,000	70
October 5	9.7	350,000	100
During low water	....	....	100
1939:			
March 10	43.4	901,000	61
May 5	42.8	954,000	70
August 28	6.5	292,000	100
During low water	....	....	100
1940:			
(No record)	....	....	....
1941:			
April 10	8.3	333,000	93
May 3	24.5	628,000	86
July 3	12.3	408,000	97
October 15	16.9	552,000	92
During low water	....	....	100
1942:			
January 14	12.8	416,000	92
April 28	36.2	911,000	82
During low water	....	....	100
1943:			
January 19	35.2	1,091,000	91
March 17	16.2	498,000	94
April 9	35.4	1,088,000	87
During low water	....	....	100
1944:			
May 23	43.7	1,124,000	84
During low water	....	....	100
1945:			
June 2	38.3	1,012,000	88

(b) Willow Cutoff

Date of observation (1)	Gage height[a] (ft) (2)	Discharge in Cutoff: Cu ft per sec (3)	Discharge in Cutoff: Percentage[b] (4)
1934:			
April 10	33.9	68,000	8
June 16	4.8	3,080	2
August 14	3.3	5,940	4
November 13	4.3	13,000	7
December 17	18.6	55,100	12
1935:			
January 26	27.1	114,000	15
March 29	42.0	298,000	26
April 30	46.8	367,000	29
June 29	45.6	433,000	33
August 16	14.4	155,000	42
October 22	1.9	73,000	52
December 17	13.6	167,000	46
1936:			
May 2	43.9	499,000	42
June 4	10.4	181,000	60
August 12	−1.0	108,000	85
September 2	−3.1	93,760	97
November 16	17.7	306,000	63
1937:			
January 27	41.9	785,000	59
February 24	55.2	1,101,000	56
July 1	20.1	427,000	81
October 9	−0.9	146,000	99
December 30	15.8	424,000	86
1938:			
April 21	41.4	865,000	70
June 9	34.5	817,000	79
September 15	2.6	204,000	100
During low water	....	....	100
1939:			
March 8	43.3	1,159,000	80
August 1	7.1	304,000	100
During low water	....	....	100
1940:			
(no record)	....	....	....
1941:			
April 12	8.6	358,000	100
May 1	25.5	707,000	92
During low water	....	....	100
1942:			
January 2	11.1	447,000	100
April 29	36.1	964,000	88
During low water	....	....	100
1943:			
January 22	34.6	1,037,000	93
During low water	....	....	100
1944:			
May 25	42.2	1,059,000	88
During low water	....	....	100
1945:			
May 31	38.4	1,105,000	96

[a] On the Vicksburg Canal gage.
[b] Percentage of the total river discharge.

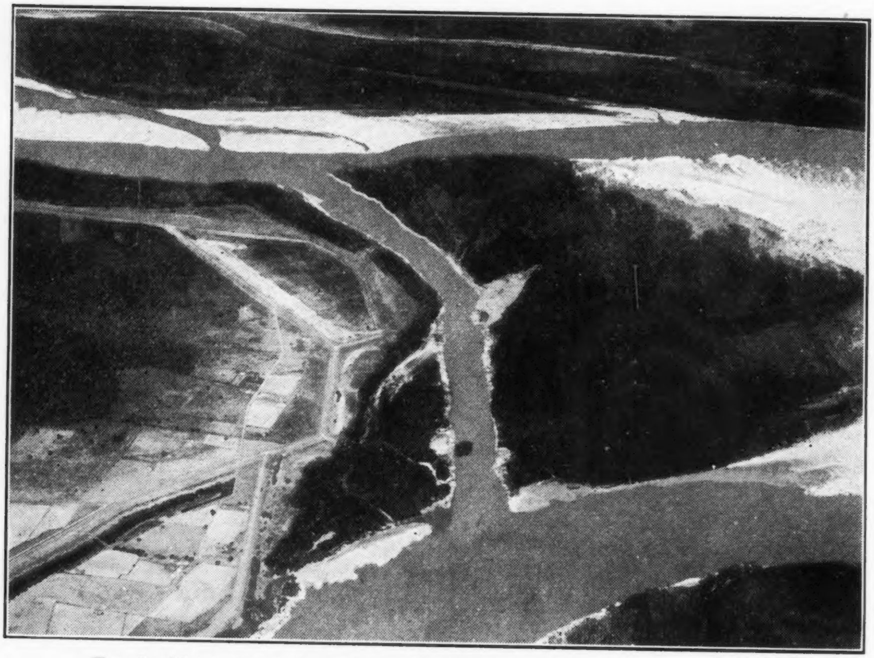


FIG. 3.—MARSHALL CUTOFF FACING UPSTREAM, LOW WATER, OCTOBER 25, 1935

FIG. 4.—MARSHALL CUTOFF FACING UPSTREAM, LOW WATER, JULY 23, 1940

Marshall Cutoff, throughout, has been a well-behaved stretch of channel and has required very little assistance. Following the high-river stages in 1934, the upper end of the pilot cut silted for a short distance to the approximate elevation of mean low water. A pump barge removed 405,000 cu yd in 12 days, thereby restoring the cutoff to a depth of 25 ft below mean low water. During 1935, 355,000 cu yd of additional shoaling required removal. Following this cleaning, the cut continued to develop (see Fig. 3) and no further dredging has been done. A view of this cutoff as it appeared on July 23, 1940, is shown in Fig. 4.

Willow Cutoff

About 1913 Willow Point (Fig. 2) was more than 7 miles long. It stuck out on the map "like a sore thumb." It had been lengthening itself year after year because of the caving habits of Albemarle Bend. During the great flood of 1913 the river made itself a new channel across the middle of Willow Point by washing out a shallow swale known as Albemarle Chute. The new river bend which thus came into existence became known as Newman Cutoff. The abandoned Albemarle Bend channel became a lake. Although the river shortened itself in this process nearly 5 miles, Newman Cutoff, according to modern terminology, amounts to a chute development and should not be dignified by the name "cutoff." The shortening was not destined to last long, for the new bend location was not favorable and it appeared that inevitably the bend would migrate again and Willow Point would probably lengthen as much as before. It was deemed advisable, therefore, to locate Willow Cutoff at a place where the resulting new channel would have no opportunity to migrate for a long time to come.

The lower end of Willow Cutoff overlaps the old Terrapin Neck Cutoff which divorced Eagle Lake in 1866. The combined effect of the two cutoffs has been to shorten the river 23.7 miles since 1866.

The method of constructing Willow Point Cutoff, in general, was similar to that employed on Marshall Cutoff, except that the greater length called for more equipment. From November 19, 1933, to January 28, 1934, seven dragline machines were operated for 322 machine days in excavating 2,392,000 cu yd, thereby lowering land elevations along the route of the pilot cut some 21 ft over a length of 13,000 ft. Between December 26, 1933, and May 15, 1934, hydraulic dredges removed 5,101,000 cu yd thereby extending the pilot cut to a depth of 14 ft below mean low water over a distance of 5,800 ft at the upper end, and to depths ranging from 10 ft to 15 ft below mean low water over other stretches. At the lower end of the cut a wide flat sand bar lay between the high bank through which the pilot channel had been dug and the river. A low slough separated this bar from higher ground and it was feared that the new cutoff channel might tend to follow this slough. It became necessary, therefore, to dredge through the bar a distance of 5,800 ft, thus making the total low-water length of the cutoff more than 20,000 ft.

Opened on April 8, 1934, nearly a month after the opening of Marshall Cutoff, on a medium high-river stage, on April 10, Willow Cutoff discharged 68,000 cu ft per sec, representing 8% of the total for the river. During the

FIG. 5.—WILLOW CUTOFF FACING UPSTREAM, LOW WATER, OCTOBER 25, 1935

FIG. 6.—WILLOW CUTOFF FACING UPSTREAM, LOW WATER, JULY 23, 1940

low-river stages that followed in June, lack of depth caused the percentages to fall as low as 2% and 4%; but, after dredging, these percentages increased appreciably during increasing river stages. The development of the discharge capacity of Willow Cutoff is best gleaned from the data in Table 1(*b*).

As was the case for Marshall Cutoff some supplementary dredging was required soon after the cutoff began to function. It was necessary to cut the bottom to 40 ft below mean low water at the center and to 22 ft at the ends. A view of this cutoff photographed on October 25, 1935, is shown in Fig. 5. Later, in June and July, 1938, a dredge was returned to the cutoff to remove a tough clay deposit in the bottom at its upper end, which had resisted the cutting action of the current and retarded development. On one other occasion, in 1936, a dredge was operated to enlarge and shape up the entrance.

In 1933, prior to making the pilot cut, the distance around Willow Point was 13.1 miles measured from the head to the foot of the pilot cut. After the 1937 flood the cutoff channel had a length of 4.7 miles measured between the same points as in 1933, indicating a new river shortening of 8.4 miles. By 1939 distances had changed so as to make the shortening nearer 7.7 miles.

At the crest of the 1934 high water, shortly after the pilot cut began to function, the hydraulic slope in the cut was 1.18 ft per mile. During the crest of the great 1937 flood, this slope had flattened to 0.53 ft per mile. In 1939 the flattening had become complete, the slope at the crest of the flood being only 0.36 ft per mile—practically the same as that of the Mississippi River in that latitude. Previous to 1937, velocities in the cutoff during high stages ran as much as from 7 to 9 ft per sec, and boats at such times preferred to travel via the bend when headed upstream. Except for these first 3 years Willow Cutoff has been easy to navigate by commercial tows. Fig. 6 is a view of this cutoff as of July 23, 1940.

TABLE 2.—Effect of Cutoffs on the 50-Mile Length of the River Shown in Fig. 2

Year	Interval (years)	Length, in Miles		Remarks
		River	Cutoffs	
1765		71±	....	
	55			Only one cutoff, the Yazoo, occurred in this period. The net effect is not known, but is presumed to have been more than 10 miles.
1820		68	61	
	62			In this interval Terrapin Neck Cutoff shortened the river 16 miles and Centennial Cutoff, 6 miles—a total of 22 miles.
1882		57	46	
	34			Albemarle Chute (known as Newman Cutoff) shortened the river nearly 5 miles.
1916		58.8	52	
	13			No cutoffs occurred in this interval.
1929		58.1	58.8	
	16			Marshall and Willow cutoffs in 1934 shortened the river nearly 12 miles.
1945		48.8	53	

River Shortening

The effect of these six cutoffs on the length of the stretch of river in Fig. 2 raised many questions, because the sum total of the shortenings amounted to

more than the total present equivalent length of river. In the course of the 80 years under consideration the lengthening processes due to river meandering offset in part the total net shortenings effected by the six cutoffs (see Table 2). For this purpose the nearly 50-mile reach from Vicksburg to Lake Providence, La., had to be considered as no statistics were available for individual parts of it.

River Steepening

Such steepening as was caused in the Mississippi River by the development of Marshall and Willow cutoffs, and by the cutoffs below, was slight and at most temporary. The river soon re-established its natural slopes in this section as may be seen from Table 3.

TABLE 3.—Flood-Crest Slopes in the Reaches Above Cutoffs

Year	(a) Upstream from Willow Cutoff to Lower End of Opposum Chute		(b) Upstream from Marshall Cutoff to Lower End of Willow Cutoff	
	Distance, in miles	Slope, in ft per mile	Distance, in miles	Slope, in ft per mile
1929	33.62	0.333	14.45	0.339
1933	33.62	0.301	14.45	0.359
1935	30.46	0.334	7.65	0.340
1937	31.98	0.319	7.65	0.315
1939	30.25	0.341	7.38	0.355

Flood-Stage Lowering

The flood-stage lowering effected in the river immediately upstream from Willow Cutoff in April, 1936 (two years after the cutoff began to function) amounted to nearly 2 ft for about a bankfull flow. Marshall Cutoff similarly caused a reduction of about 1 ft. These data are based on the reduction in fall observed within the cutoffs.

Actually, the total amount of flood-stage lowering in this part of the river is on the order of that observed on the Vicksburg gage—that is, from about 7 ft to 8 ft, varying with stage conditions. The total lowering is transmitted upstream from the five cutoffs below Vicksburg, and represents a cumulative effect to which these cutoffs and various dredging operations have contributed, minus losses caused by the inability of the stream bed to lower itself in places. Upstream from Vicksburg seven cutoffs (including Marshall and Willow), plus channel improvements, increase the stage lowering progressively until on the Arkansas City, Ark., gage their cumulative effect amounts to a decrease of from 12 ft to 13 ft in flood stages as compared to equivalent river discharges in 1933.

Flood-stage reductions have been of inestimable benefit to backwater farmers; they have released millions of dollars for raising levees; and, by dispensing with a floodway on the west side of the river, they have saved a large area of productive lands from needless overflow. Furthermore, floodwater travel has become accelerated. In the reach from Arkansas City to Natchez alone, the time of travel of flood crests has been shortened several days.

The cutoff program was inaugurated in 1932 as part of the work of stabilizing the navigable channel of the Mississippi River. The first artificial cutoff to

be opened was at Diamond Point below Vicksburg on January 8, 1933. The cutoff program progressed rapidly, as may be noted from Table 4. By 1939, thirteen river bends, totaling 156.5 miles, had been abandoned between the mouth of the Arkansas River and Baton Rouge. The aggregate length of the thirteen new channels across the necks amounted to 40.7 miles, thus accounting for a net shortening of 115.8 miles, measured at bankfull stage, in what formerly was a 462-mile river distance.

In 1946 there were sixteen cutoffs in operation, three having been constructed in 1941 and 1942 between the mouth of the Arkansas River and Memphis. The sixteen cutoffs together have shortened the Mississippi River 151.9 miles between Memphis and Baton Rouge; but, in addition, other river improvements (notably chute enlargements accelerated by dredging operations) have created further shortenings, decreasing the total length 170 miles between Memphis and Baton Rouge. This value represents the length by which the low-water navigation channel between these two cities has been shortened. In 1932 the river distance between Memphis and Baton Rouge was about 680 miles.

TABLE 4.—CUTOFFS ON THE MISSISSIPPI RIVER BETWEEN MEMPHIS, TENN., AND BATON ROUGE, LA.

(Location and Distances as Determined by Approved Center Line and Scaled Mid-Mean Low Water Mileage)

Name	Opened	Location of upper end (Mile AHP[a])	Distance in Miles as of Date Cutoff Was Made			Length of dredged cut (in ft)	Fall Across Neck (In Ft)	
			Across neck	Around bend	Net shortening		At high water 1929	At mean low water[b]
Hardin	March 18, 1942	676.0	1.86	18.82	16.96	5,900	4.0[c]	8.1
Jackson	April 26, 1941	624.4	2.40	11.06	8.66	11,300	4.5[d]	3.6
Sunflower	February 16, 1942	621.9	2.43	12.87	10.44	10,300	4.0[d]	4.3
Caulk	May 13, 1937	568.5	2.0	17.2	15.2	4,400	2.9[e]	5.8[f]
Ashbrook	November 19, 1935	542.0	1.9	13.3	11.4	4,530	5.4	2.8
Tarpley	April 21, 1935	535.0	3.6	12.2	8.6	13,000	1.8	3.4
Leland	July 8, 1933	531.7	1.4	11.2	9.8	....[g]	4.3	3.4
Worthington	December 25, 1933	507.0	3.8	8.1	4.3	17,600	3.7	1.6
Sarah	March 23, 1936	498.2	3.2	8.5	5.3	12,600	2.2	3.0
Willow	April 8, 1934	457.7	4.7	12.4	7.7	22,000	4.0	3.7
Marshall	March 12, 1934	443.9	3.1	7.3	4.2	13,600	2.4	2.2
Diamond	January 8, 1933	420.0	2.6	14.6	12.0	9,175	2.2	4.2
Yucatan	Fall of 1929	404.4	2.6	12.2	9.6	....[g]	3.7	2.8
Rodney	February 29, 1936	385.4	4.1	9.9	5.8	13,000	2.7	2.0
Giles	May 25, 1933	363.8	2.9	14.0	11.1	10,000	4.6	2.8
Glasscock	March 26, 1933	342.1	4.8	15.6	10.8	20,800	3.2	3.4
Total	....	....	47.4	199.2	151.8	168,205	55.6	57.1

[a] "Above heads of passes," the starting point for river distances established in 1942. [b] As of date of opening cutoff. [c] At bankfull high water in 1937. [d] At bankfull high water in 1940. [e] In 1937 the high water fall was 4.7 ft. [f] Actual fall on August 28, 1936 was 10.6 ft, at low water. [g] Natural cutoff; no artificial channel was dredged.

In view of the undesirable aftereffects which have repeatedly been experienced at artificial cutoffs made on other rivers, concern has been expressed regarding the wisdom of launching an extensive cutoff program on the Mississippi River. Prior to the beginning of the present development, a study was made

of the difficulties and liabilities experienced at cutoffs elsewhere, and a clear understanding was gained regarding the pitfalls to be avoided. In the first place, it became clear that the river should not be straightened unduly. To straighten it deliberately in time would have tended to convert the Mississippi River into a shallow stream 3 or more miles wide, interspersed with many islands and bars separated by interlacing channels. Such a "braided" river would have impaired navigability to the point of forcing its abandonment; yet a perfectly straight channel alinement from Cairo to the Gulf of Mexico has not been without its proponents.

The conception also existed that water flowing through a cutoff raised stages downstream by "piling-up." Instances were cited where this had been actually witnessed in early days when the river broke through a narrow neck of land during a high stage. It was important, therefore, to avoid inviting these "piling-up" effects and this result was readily accomplished through the adoption of the pilot-cut principle as previously explained.

Another "bête-noire" in cutoff procedure—namely, loss of valley storage—had to be avoided as much as possible. The retirement of a series of large river bends by cutoffs in time obviously would rob the river of millions of acre-feet of useful valley storage, and would tend to raise flood stages at points downstream. This problem was solved by leaving the retired bend-way channels wide open—thus permitting floodwaters to enter them, flow around the old bends, and flood the land back to the levees as in the past. The preservation of this valley storage, although diminished somewhat by a reduction in flood stages, proved to be a wise forethought on the part of General Ferguson. The valley storage remaining available still is vast and constitutes an important asset in the general plan. At cutoffs made as early as 1933 or 1934, which have since become full-size Mississippi River channels, water still continues to flow through the abandoned bend-way channels during high stages and may continue to do so for an indefinite number of years.

Summary

The cutoff history of the 50-mile reach described was selected as the topic of the paper because it brings out strikingly the vast difference between the art of cutoff procedure as originated by General Ferguson, and nature's unrestrained art. It is the difference between a conservative, gradual process, at all times under complete control, and a cataclysmic way of producing a new channel in a matter of days.

Marshall and Willow cutoffs are examples of safe engineering practice applicable to large meandering rivers, but furnish no criterion for problems of this kind in small streams. The success and simplicity of the work at Marshall and Willow cutoffs must not be taken, however, as representative of all Mississippi River cutoff operations. In many instances, adverse physical conditions—some of them natural, others man made—called for considerable departure from the methods and principles employed at Marshall and Willow, and in some cases these departures affected, in large part, the cutoff project as a whole.

The original plan contemplated opening the principal cutoffs downstream from Vicksburg ahead of those planned at points upstream. This was for the

purpose of developing, in the lower river, adequate capacity for transporting the increased sediment load that was expected to result from upstream operations. The plan of procedure has remained fundamental, but it was marred in a few instances by unforeseeable events. Thus, Leland Cutoff, at the foot of the famous Greenville Bends, had to be constructed more than a year ahead of schedule because of an unexpected break in Leland Neck during the summer of 1933. Adventitious as regards both time and place, Leland Cutoff forced a complete revision to be made of the original rectification plan for the Greenville Bends. Moreover, Leland Cutoff had severed an extremely narrow neck, which was contrary to the principles laid down. In the revised plan it became necessary to place a cutoff at the head of the Greenville Bends across Ashbrook Neck—another perilously slender neck. Complex problems have resulted from this situation which in turn have called for large expenditures in dredging and revetment work. Another cutoff that had to be constructed a year ahead of schedule was located across Worthington Point. Here, a rich agricultural area on the east bank had demanded relief from the rapid bank caving in Kentucky Bend, and hope had been entertained that the cutoff might alleviate this threat by causing the river to shift its main channel over to the west bank.

Contrary to plan, also, was the exceedingly slow development of the cutoff farthest downstream—Glasscock, the second artificial cutoff to be opened. In 1938, 5 years after it was opened, despite repeated dredging, Glasscock Cutoff was carrying only one third of the total river discharge. Resistance to erosion on the part of the thick mantle of claylike alluvium which prevails in the southern part of the valley was responsible for retarding its development. Agreeably contrary to expectation, however, little or none of the sand liberated at upstream cutoffs and other rectification operations appear to have affected Glasscock Cutoff. It is presumed that this sand, in large part, was deposited by the river in the abandoned bends which purposely had been left open to serve as sand depositories.

In conclusion it is desired to emphasize that the principal objective of the cutoff program has been to lower flood stages, and that the lowerings that have been so effected have fully met the original estimates made in 1930. In future years, when river bends lengthen into new loops, additional cutoffs will doubtless be required to maintain the flood-stage lowerings effective as of 1946. The rate at which such new cutoffs will be needed of course will depend upon the rate at which river lengthening takes place, and cannot be forecast reliably. The 1944 plan [3] of stabilizing riverbanks by applying suitable revetments to resist bank caving, should be of material assistance in postponing the time when additional cutoffs will be needed in the stretch of river shortened by the sixteen cutoffs listed in Table 4.

Acknowledgment

The photographs in this paper were taken by the U. S. Army Air Corps and were made available, for use in this paper, by the Mississippi River Commission.

[3] "New Project for Stabilizing and Deepening Lower Mississippi River," by Charles Senour, *Proceedings*, ASCE, February, 1946, p. 145.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

STABILITY OF SOIL SLOPES

By Ek-Khoo Tan,[1] Jun. ASCE

Synopsis

All cohesionless soils assume an angle of repose. On the other hand, when the soil possesses some cohesion the natural slope can be made steeper. A study of the phenomena and mechanics involved in the failure of earth slopes is reported in this paper. In the model studies of sand slopes used to investigate the slide phenomenon, the angle of repose of cohesionless soil was found to be an entirely superficial phenomenon and independent of the height of the slope. However, the inclination and the height of a bank of cohesive soil were found to depend upon cohesion. A technique to create an equivalent cohesion in a cohesionless soil was developed to reproduce actual sliding failures on a small-scale model. These laboratory failures were found to be remarkably similar to those observed in nature; and they proved ideal for studying the mechanics of failure.

Photoelastic studies were made on gelatin models to investigate the distribution of stresses, and their relative order of magnitude. There was a remarkable resemblance between the pattern of shearing stress and the pattern of shearing strain obtained by means of the sand model. It appears that failure is initiated in a plastic region near the top of the slope.

An approximate mathematical solution of the slope problem based on the theory of plasticity was developed to determine the possible position of the sliding curve and the stress conditions leading to failure. The theory shows the slip lines to be arcs of circles. A sliding failure is imminent whenever the height and inclination of slope, and the shearing properties of soil, bear such a relation to each other that a plastic region develops and tends to enlarge progressively.

Introduction

One of the most important, and at the same time most interesting, problems in the field of soil mechanics is the investigation and analysis of the stability of

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by June 1, 1947.

[1] Asst. Chf., Aerodromes, Air Routes and Ground Aids Section, Provisional International Civil Aviation Organization, Montreal, Que., Canada.

soil slopes. Whenever a natural bank of cohesive soil is exposed and left unsupported laterally, experience has shown that there are certain critical height-to-slope relations necessary for stability. When these conditions are exceeded, a failure by sliding is inevitable. This slide phenomenon has been observed extensively by engineers, and has been found to follow typical forms of failure.

The most complete analysis of landslide phenomena was conducted by the Swedish Geotechnic Commission, whose investigations were prompted by the failure of a quay wall in Göteborg in 1915. Subsequently, the commission made a complete study of slides in railway cuts, and found that the shape of the sliding curve in cohesive materials showed a decided curvature that could be approximated by an arc of a circle. The expression of the fundamental principles of the "Swedish circular arc methods" are credited to K. E. Petterson[2] and S. Hultin.[2] In 1927 W. Fellenius[3] and H. Krey[4] perfected the method for practical application. Subsequently, contributions were added by Karl Terzaghi,[5] M. ASCE, J. Jaky,[6] and D. W. Taylor,[7] Assoc. M. ASCE.

Part I of this paper concerns the phenomenon of slides and the mechanics of failure, which were investigated by means of small-scale model slopes of soil. Part II concerns the photoelastic investigations made on gelatin models in order to obtain a clearer concept of the distribution, and of the relative order of magnitude, of the shearing stresses within a slope as the failure condition is approached. Part III is devoted to the development of an approximate mathematical solution of the slope problem based on the theory of plasticity, which permits the determination of the possible position and form of the sliding curve and of the approximate stress conditions leading to failure.

I. Investigation of Soil Slopes by Models

Model tests of soil slopes were made to study the mechanics of failure in cases where the slide phenomenon is controlled by gravitational influences only. In cohesionless sand the angle of repose represents a limiting condition when the soil assumes a natural stable slope. When the soil possesses some cohesion, on the other hand, it is capable of standing at a considerably steeper slope than that of a cohesionless material. The maximum inclination and height to which a stable earth slope can be constructed depend primarily on the character and physical properties of the earth in the slope, and on the moisture conditions.

The apparatus used for the model investigation on sand (see Fig. 1) consisted of a galvanized iron box with tight joints and an open end. This box, placed on a firm support, was provided with a device for tilting it to any desired angle. The angle of slope of soil was measured by a clinometer. A "coarse to fine" sand was used throughout the investigation.

---

[2] Statens Jarnvagers, Geotekinson Comm., Slutbetankande, May, 1922.

[3] "Erdstatische Berechnungen mit Reibung und Kohäsion," by W. Fellenius, Ernst, Berlin, 1927.

[4] "Erddruck, Erdwiderstand und Tragfähigkeit des Baugrundes," by H. Krey, Ernst, Berlin, 1932.

[5] "The Mechanics of Shear Failure on Clay Slopes and Creep of Retaining Walls," by Karl Terzaghi, *Public Roads*, December, 1929.

[6] "Stability of Earth Slopes," by J. Jaky, *Proceedings*, 1st International Conference on Soil Mechanics and Foundation Eng., Harvard Univ., Cambridge, Mass., Vol. II, 1936, Paper G-9, p. 200.

[7] "Stability of Earth Slopes," by D. W. Taylor, *Journal*, Boston Soc. of Civ. Engrs., July, 1937.

1. Angle of Repose Phenomenon for Dry Cohesionless Materials.—A series of tests was made with the sand deposited in two density states, loose and dense. In each case the slope was formed to its natural angle of repose of 33°, which was determined by a clinometer. A very loose state was obtained by spreading sand in thin layers with a large funnel, the thickness of the layer being controlled by the height of a funnel tip above the sand. Quite a dense state was

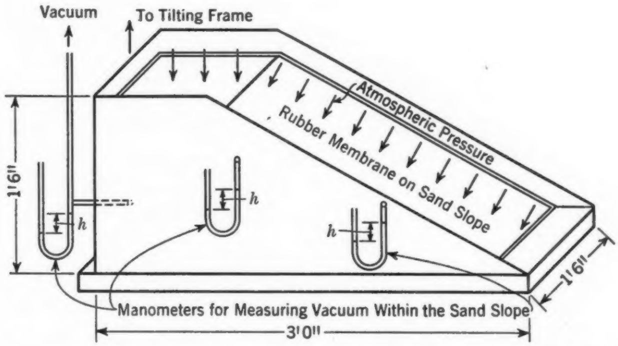


FIG. 1.—MODEL APPARATUS FOR INVESTIGATING THE STABILITY OF SAND SLOPES

obtained by using a vibrating device which compacted the sand to the desired density. The technique of placing the sand was developed so as to obtain reproducible consistent results. The apparatus was then tilted slowly until the first visible movement of the sand was observed. The angle of repose of dry sand used in these tests was found to vary between two limiting values—a maximum of 36° just before a slide, and a minimum of 33° just after a slide. The lower angle, which occurs after a slide, is created by the momentum of the material as it slides down the slope in an effort to seek a stable position. Failure was observed to begin at the top of the slope and was confined entirely to the surface layers of the slope. Because of the superficial nature of the slide phenomenon, the angle of repose of dry material is independent of the height of the slope; the slope always tending to flatten itself by superficial slides to the angle of repose. The results in Table 1 for a series of tests on the given material, compacted at different densities, show that the angle of repose is practically independent of the density of the material.

TABLE 1.—ANGLE OF REPOSE ϕ, IN DEGREES; AVERAGE OF A SERIES OF TESTS FOR EACH MATERIAL

Material	Void ratio e	Maximum ϕ	Minimum ϕ
Sand:			
Loose	0.73	36	33
Compacted	0.65	36	33.5
Loose Gravel:			
⅜-in.	...	36.5	31
¾-in.	...	42.5	27

To determine the effect of the coarseness of the material on the angle of repose, tests were made on gravels separated into ⅜-in. and ¾-in. sizes. The maximum angle of repose for a ⅜-in. material (36.5°) is only slightly larger than sand, whereas for the ¾-in. material the angle of repose (42.5°) is noticeably

larger. This is due to the greater interlocking effect as the particle size increases. Angles of repose of 45° are a frequent occurrence in talus slopes of coarse fragments. The angle of repose of 31° after a slide, for the $\frac{3}{8}$-in. material, is noticeably smaller than that for the sand; and for the $\frac{3}{4}$-in. material the angle of repose (27°) is still smaller, the momentum effect being more pronounced for the larger particles in flattening the slope.

2. Phenomenon of Slides in Slopes of Cohesive Soils.—When a slide occurs in a cohesive soil, it is a deep-seated rather than superficial phenomenon involving the rupture and the breaking away of a large mass of earth from the slope. The maximum inclination and height to which a stable slope of cohesive soils can be constructed is definitely limited, and depends primarily on the magnitude of the cohesion that the soil possesses.

The cohesion in natural soil deposits is the result of an internal system of forces: (1) Capillary forces arising from capillary moisture films at the grain contact; and (2) intermolecular attractions between the finer soil grains.

According to the usual concept of model analysis there should first be a geometric similarity between model and prototype, and the forces controlling the phenomenon should have a constant ratio. These requirements cannot be satisfied readily, because if the particle size of the soil is reduced in accordance with the scale ratio, the properties of the soil are altered materially. Gravitational and molecular forces cannot be altered. Capillary forces are difficult to control in the laboratory in a small-scale test. This means that every model test is really full scale. However, in a small-scale model, if a slide phenomenon can be produced similar to slides observed in nature, then it can be stated that the performance of the model is similar to nature.

In order to reproduce this phenomenon in the small-scale model it was necessary to develop a special technique which, because cohesion is the important factor, involved creating an equivalent cohesion in a cohesionless material that could be controlled so as to satisfy model requirements and reproduce a slide failure in the laboratory. However, it must be remembered that the influence of the mass of the soil involved in a failure cannot be learned from a small-scale model; but the model does afford a good insight into the mechanics of failure.

Cohesion is essentially an intergranular stress phenomenon created in one case by capillary forces arising from the capillary moisture film at the grain contact or by molecular forces between the finer soil grains, which resist shearing displacements of the grains. Thus, natural cohesion is the product of an internal force times the tangent of the angle of friction between grains. In the technique described herein an identical situation can be created by substituting a uniform system of external force for these natural internal systems of forces which are the source of natural cohesion.

The equivalent cohesion in the model soil slope was obtained by covering the cohesionless sand slope by a rubber membrane. This was stretched loosely over the slope but was tightly clamped to the sides of the apparatus, as shown in Fig. 1, so that the air in the soil mass could be evacuated. The soil was thus subjected to a uniform external pressure which could be maintained constant and be easily controlled or varied at will. This uniform external pressure pro-

duced an essentially uniform intergranular stress throughout the soil mass. The most important advantage was that the cohesion could be made sufficiently small so that a failure by a slide could actually take place.

Model slopes were formed: (1) With sand placed in a very loose state, and (2) with sand placed in a medium dense state. The sand was then transformed to a soil possessing an equivalent cohesion. The applied uniform system of external forces was determined by measuring the vacuum within the soil mass by means of three water manometers in Fig. 1, placed at intervals along the slope. The fact that they gave equal readings showed that the pressure was practically uniform over the slope. The apparatus was then tilted until a failure occurred. The angle of slope was measured; the form of failure was observed

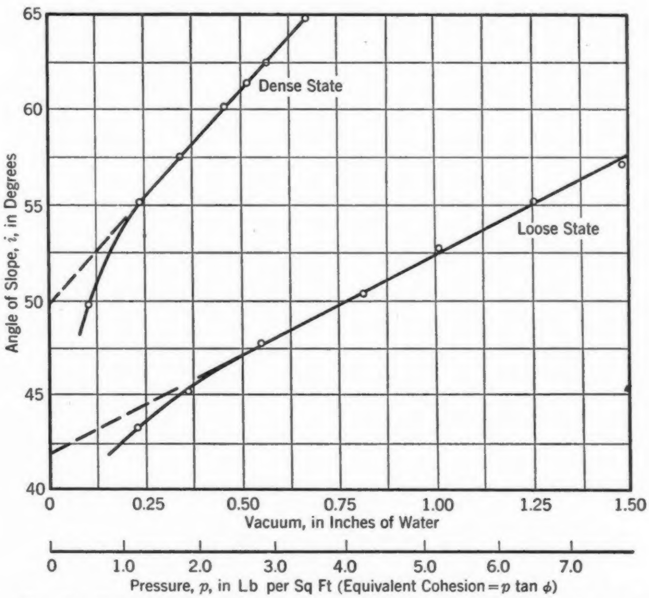


FIG. 2.—RELATION BETWEEN THE ANGLE OF SLOPE AT FAILURE AND THE MAGNITUDE OF THE EQUIVALENT COHESION

after the rubber membrane was removed. A new slope was built for each test. Fig. 2 shows the relations between the equivalent cohesion and the resulting angle of the slope at failure for the loose and dense states. It is important to note that: (*a*) The angle of slope at failure increases linearly with the equivalent cohesion except for very small value where the restraining influence of the rubber membrane itself is appreciable compared with the air pressure; and (*b*) the density of the sand has a very marked effect on the angle of slope at failure, because of the important increase in the angle of friction with increase in density.

3. Model Investigations of the Deformations Within the Soil Slope and the Location and Form of the Rupture Surface.—A procedure was devised to obtain a record of the deformation within the soil slopes, and the locations and forms of the rupture surface. A piece of glass, which was cut to the shape of the sand

slopes and carefully ruled with vertical lines of lampblack and oil, about 1/16 in. wide (Fig. 3), was placed within the soil mass. After the glass had been placed against the side of the apparatus, the sand was carefully deposited against it,

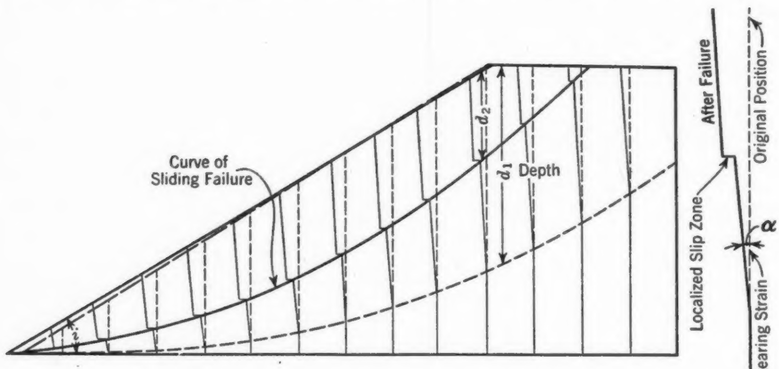


FIG. 3.—SHEARING DEFORMATIONS AND SLIDING CURVE DEVELOPED IN THE SAND SLOPE AT FAILURE

and the slope was formed as before. When any deformation or sliding occurred within the soil mass, the movement of the sand was registered on the glass by the displacements of the vertical lampblack lines. These movements were observed after the sand had been carefully removed and the glass plate was withdrawn. Displacements of about one fifth of the width of the line could readily be detected. Fig. 3 illustrates a typical curve of failure. These investigations reveal the following important facts, which are shown graphically in Fig. 4:

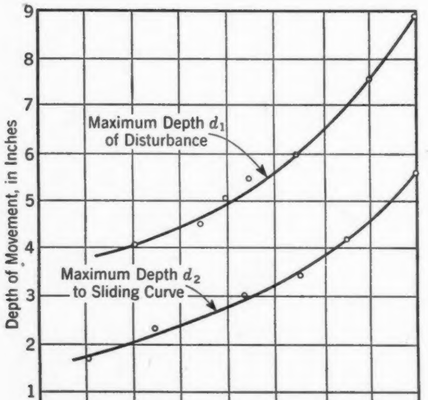


FIG. 4.—RELATION OF THE DEPTH OF MOVEMENT TO THE SLOPE ANGLE i AT FAILURE

(1) As cohesion increases, a larger mass of soil is involved in the failure and the principal failure curve recedes deeper from the surface of the slope;

(2) In all cases the zone of disturbance extends a considerable distance below the final sliding curve; and

(3) An appreciable volume change was observed in the soil slope as evidenced by the vertical movements of the surface of the slope. A subsidence occurred in the case of loose sand, and an expansion in the case of dense sand.

The angular displacements of the lines, or shearing strains, as shown in Fig. 3, reached a maximum value of about 7°. This represents a shearing strain of 0.1225 in. per in.

A study of the displacements within the soil slope was made at different stages prior to failure. These measurements establish the fact quite clearly that failure occurs in a soil slope after the strain exceeds a certain rather well-defined maximum value.

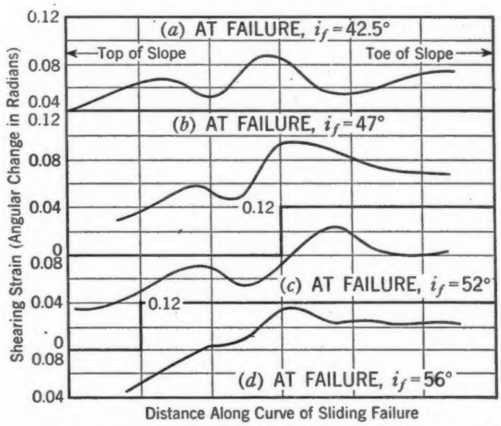


FIG. 5.—DISTRIBUTION PATTERN, SHEARING STRAINS ALONG THE ADJUSTED CURVE OF SLIDING FAILURE, FOR VARIOUS ANGLES OF THE SLOPE i_f AT FAILURE

This maximum strain, which is associated with first evidence of an incipient failure by a slide, seems independent of the angle of slope and occurs at or near the top of the slope. The curves of Fig. 5 show the distribution of shearing strains along the sliding curve at impending failure. The wave pattern is quite consistent in all tests, with a maximum near the top of the slope. This peculiar wave propagation, or development of shearing strains, in the soil slope at impending failure bears a close resemblance to the variation of maximum shearing stress obtained in the photoelastic studies on gelatin models shown subsequently in Fig. 9.

4. Investigation of the Failure of Model Vertical Banks.—It is a well-known phenomenon in nature that cohesive soils can stand up in a laterally unsupported vertical bank. In these tests the initial equivalent cohesion was made much greater than required for the stability of the bank of the given height. The cohesion was then decreased slowly until the bank failed.

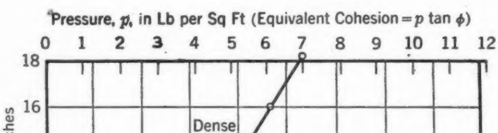


FIG. 6.—RELATION BETWEEN THE CRITICAL HEIGHT OF A VERTICAL BANK AND THE MAGNITUDE OF EQUIVALENT COHESION

The results of the investigation (see Fig. 6) show that the critical height at which the laterally unsupported embankment will stand vertically is a linear function of the cohesion. When failure occurred, the break at the surface of the embankment was found to be approximately 37.3% of the critical height of the embankment. This agrees quite closely with actual failures observed in nature where breaks occur at 0.4 h_c to 0.5 h_c (see critical height h_c of bank in Fig. 6(*a*)).

5. Analysis of the Curves of Failure by the Swedish Circular Arc Method.—A series of the observed curves of failure was analyzed by the usual Swedish

circular arc method. The principles of this method are based on the following assumptions: (1) The problem is a two-dimensional one; (2) failure takes place along an arc of a circle; (3) the mass of soil above the failure curve tends to rotate about some center of rotation; and (4) the shearing resistance is fully mobilized along the curve of sliding failure.

In the standard method of analysis the mass of soil above a sliding curve is divided into a number of elementary vertical sections and the usual principles of static equilibrium are then applied; that is, $\Sigma M=0$; $\Sigma H=0$; and $\Sigma V=0$. In this case ΣM is taken as the moment, about the center of the circular arc, of the resisting forces along the sliding curve which moment, for stability, must balance the moment of the forces causing sliding. The lateral earth-pressure forces acting against the vertical side of each element are considered as conjugate stresses and the resultant is zero. The weight of the soil element acts through the centroid of the element and is resolved into its tangential and normal components, W_t and W_n, respectively, on the sliding curve. In Fig. 7 let the material above the critical slide plane be divided into sections 1 to 9, exerting forces or weights W_1, W_2, etc. Denote the respective components of weight by W_n (normal) and W_t (tangential), with numerical values as follows:

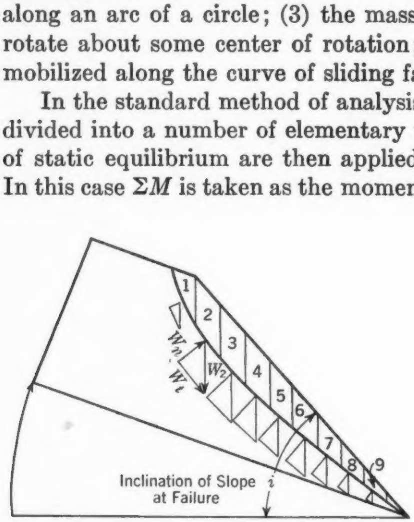


FIG. 7.—SWEDISH CIRCULAR ARC ANALYSIS

Section	W_n	W_t
1	1.00	1.75
2	2.75	3.00
3	3.50	3.00
4	3.25	2.20
5	2.80	2.00
6	2.70	1.60
7	2.20	1.50
8	1.75	1.20
9	1.00	0.70
	20.95	16.95

The tangential component W_t is the force that tends to cause sliding. The normal component W_n tends to resist sliding by mobilizing friction along the sliding surface, which, according to the usual concepts of sliding friction, can become equal to $W_n \tan \phi$ as a limiting value. The ratio of the forces resisting sliding to those causing the sliding is usually defined as the factor of safety. According to this concept this factor of safety should decrease to unity at impending failure.

The observed curves of failure, obtained from the tests on cohesive soil slopes, were analyzed and the results showed that, the angle of internal friction

ϕ being 33°, the factor of safety at failure was $\frac{\Sigma W_n}{\Sigma W_t} \tan \phi = \frac{20.95 \times 0.649}{16.95}$ $= 0.80$; that is, somewhat less than unity. The significance of a factor of safety less than unity at failure is not fully understood. The real mechanics of failure are evidently not fully taken into account by the simplified Swedish circular arc method. The circular arc method assumes an average condition along the entire length of arc. The actual stresses set up in the soil slopes probably bear very little resemblance to those assumed in the circular arc method.

II. Photoelastic Studies of Gelatin Models

Photoelastic studies were made on gelatin models of slopes to obtain a clearer picture of the distribution, and the relative order of magnitude, of the shearing stresses set up within a slope. Such studies offer great possibilities for investigating the distribution of shear stresses and the stability conditions of soil embankments. Since the problem was to determine the stresses, in the soil slope, created by gravity forces, it was necessary to use a photoelastic

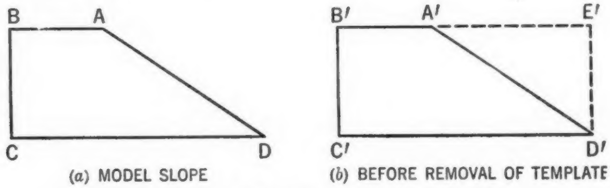


Fig. 8.—Similitude Relations in a Gelatin Model

material sensitive enough to show the stresses set up by its own weight. Gelatin seems to be quite satisfactory material for this purpose. The technique of making gelatin models has been fairly well developed.[8,9,10] Gelatin, glycerin, and water were mixed in the proportion 12:14:76. Glycerin was added to produce a tougher and more elastic gel. The higher the concentration of the gelatin, the more sensitive the gel becomes. The properties one should look for in a gel are a high degree of translucency, optical sensitivity, and elasticity. It was found that a few drops of hydrochloric acid added to the mixture would yield a more translucent model.

A glass flume, 17 in. by 10 in. by $3\frac{5}{8}$ in., with 3/16-in., removable, plate-glass front walls and back walls (see Fig. 8) served as a container for the gelatin model. A perforated templet A′ D′, made of galvanized iron and shaped to simulate the desired slope, was placed in the container. The perforations in the templet permitted the gelatin to be poured so that a continuous medium with a level upper surface was formed while the gelatin was cooling and setting. The gelatin model slope could then readily be cut to shape after the gelatin had set.

[8] "Gelatin Models for Photoelastic Analysis of Stress in Earth Masses," by F. B. Farquharson and R. G. Hennes, *Civil Engineering*, April, 1940, p. 211.

[9] "Gelatin Models," by T. R. Cuykendall, Eastern Photoelasticity Conference, May, 1939.

[10] "The Application of Photoelasticity to Soil Mechanics," by R. G. Hennes, 1st Northwestern Photoelasticity Conference, March, 1940.

With this perforated templet in place, and before the upper part A′ E′ D′ above the templet had been removed, the stress distribution in the gelatin solution was evidently hydrostatic. The distribution remains practically unchanged when the gelatin sets, as shown by the fact that there was no double refraction when the gelatin block was put into the polariscope.

To prevent the gelatin model from sticking to the glass side of the flume, and to eliminate friction, cellophane was attached to the glass side by a thin layer of vaseline. It was found necessary to allow the gelatin to cool to the same temperature as the flume before it was poured, to prevent the cellophane from wrinkling and to avoid disfiguring the surface of the model.

After the solution was poured into the flume, it was then allowed to set for a period of at least 30 hr, but not more than 48 hr, the period in which the shrinkage of the gel becomes excessive. The flume was placed on its side, the glass plates were removed carefully, and the cellophane was peeled off. The material above the templet was discarded and the templet itself was severed carefully from the model. The gelatin and the inner face of each glass plate were lubricated with thick mineral oil and the glass plate was replaced. This procedure was followed meticulously on both sides of the model. When the model, now shaped to the desired slope, was returned to its vertical position, the body forces came into play, causing the gelatin to deform.

1. Similitude Relations of Gelatin Models.—In making gelatin models certain conditions should be satisfied: (1) Surfaces AB and AD in Fig. 8 must be free of stress for the natural slope; and (2) the stresses in the slope should be confined to those caused by gravitational forces only.

In the gelatin model the surfaces BC and CD, while simulating conditions in the model sand slope, introduce a rigid boundary which does not exist in natural slopes. However, at failure, when a plastic state exists within the region of failure, the effect of the rigid boundaries may become less important. To reproduce the desired boundary conditions in the gelatin an ideal procedure would require that the gravitational force be applied and released alternately. However, since this is not possible, the aforementioned experimental procedure, involving the templet, was used. Before the perforated templet (A′ D′ in Fig. 8) was removed, the block of gelatin A′B′C′D′ was under a hydrostatic stress distribution. When the templet and wedge A′E′D′ are removed, the free boundary condition along plane A′D′ is satisfied, the free boundary condition along A′B′ is unchanged, and the gravitational body force in the volume A′B′C′D′ is brought into play.

2. Distribution of Maximum Shearing Stresses; Isochromatics.—If a transparent model of gelatin is stressed in its plane, and, if linearly polarized light is sent through the stressed body by means of Nicol prism, interference phenomena and colored fringes called isochromatics are produced which can be observed on a screen by the use of a second Nicol prism. On the screen these isochromatic lines will appear in repetition, each color corresponding to a certain principal stress difference:

$$(\sigma_1 - \sigma_2) = \text{constant} = 2\,\tau_{max} \qquad (1)$$

in which σ is the unit principal stress; and τ is the shear stress. The iso-

chromatics gave a clear picture of the distribution and the order of magnitude of the maximum shearing stress throughout the slope in the gelatin model. The outside surface of one glass plate was divided into 3-in. squares, because the lenses of the polariscope allowed only a field of 5 in. in diameter to be observed. When the entire field for one position of the model had been mapped, the angle of tilt was increased and the isochromatic pattern was again mapped. The tilting caused a slight variation in the configuration and intensity of the maximum shearing stresses.

Two gelatin models were made and showed identical patterns of isochromatics. A calibration of the gelatin was made by cutting off a right prism 3 in. by 3 in. by 6 in., and the fringes resulting from its own weight were measured. It was found that the calibration constant for the fringe order corresponded to 84% of height, h, of the prism. This calibration permits an approximate quantitative study of the maximum shearing stresses. The values of the maximum shearing stresses, as determined on the gelatin model, were transformed to the corresponding values in the sand model by multiplying the calibration constant for the maximum shearing stress values obtained on the gelatin model by a scale ratio:

$$\frac{\gamma_s}{\gamma_g} \times \frac{L_p}{L_m} = \gamma_r \, L_r \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which γ_r is the scale ratio of the unit weight of soil (γ_s) to the unit weight of gelatin (γ_g) and L_r is the linear scale ratio of the sand phototype (L_p) to the gelatin model (L_m).

To study the possible conditions with respect to the boundary BC and CD, Fig. 8, which are likely to exist in the sand model or in the prototype slope, photoelastic model studies were made for two conditions—with an adhering and a lubricated base. By separating the gelatin along boundary BC the tension in the material was eliminated at this boundary. When the model adheres to the base, there is a concentration of stress at the toe of the slope; but, when the base is separated and lubricated, the stress concentration at the toe disappears and there is also a general decrease in stress throughout the model as shown in Fig. 9(a). The separation of the gelatin from the boundary BC decreased the stresses in the vicinity of this boundary, but increased the stresses near the toe of the slope. These represent possible limiting conditions. Judging from the stress and strain patterns, where one fringe order $= 0.84\,h$ $= (\sigma_1 - \sigma_2) = 2\,\tau$, it is probable that the actual stress condition in the sand lies somewhere between these two limiting conditions, since the shearing stresses along the rigid boundary at the toe of the sand slope, where the height is small, must also be relatively small.

The gelatin model was formed to an initial slope of 29°. The distribution of shearing stresses formed a regular pattern within the slope and increased in intensity as the angle of slope was increased by tilting successively to 34°, 39°, 44°, and 49°, as shown in Fig. 9. To interpret these stress conditions in terms of mechanics of failure in the sand slope, contours of equal shearing strength have been superimposed on the isochromatic patterns in Fig. 9. The positions of these lines were determined by means of the scale ratio of Eq. 2. The cali-

bration constant (0.84% of height, *h*, of gelatin for one fringe order) is substituted in Eq. 2. The lines are always parallel to the surfaces of the slope.

It is important to note that the wave pattern of shearing stress bears a close resemblance to the pattern of shearing strains in Fig. 5 obtained in the model investigations on slopes of sand possessing equivalent cohesion. The significance of the wave pattern in Fig. 5 is now made clear. In the same gen-

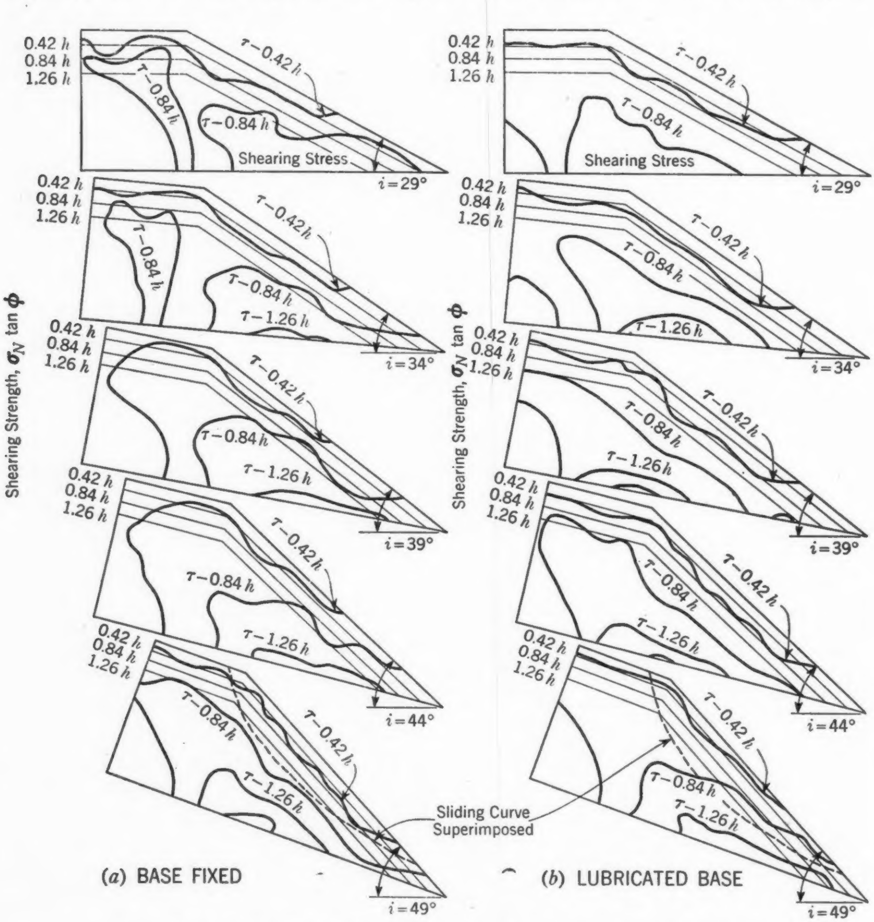


Fig. 9.—Isochromatic Patterns for Difference Values of the Slope *i*

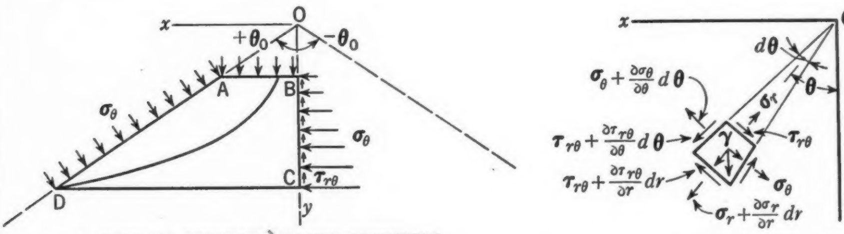
eral region where the shear-stress contour lies above the corresponding contour of shearing strength, the shearing strains are a maximum, and the material has reached a plastic state. Physically the shearing stress cannot exceed the shearing strength; but the excess stress must be transferred to the adjacent regions where there is a reserve of strength. Thus the plastic state encroaches on the elastic region and spreads progressively until, finally, a rupture surface develops and the slope fails by a slide.

It is now evident that, at 29°, a sand slope is stable. As the slope is tilted to the angle of repose or slightly greater (34°) a plastic region develops only near the surface of the slope, which accounts for the superficial nature of the angle-of-repose phenomenon.

When the angle of slope is to be increased, the equivalent cohesion must be increased, accordingly, to some higher value in order to maintain a stable slope. The uniform cohesion, in effect, raises the line of constant shearing strength with respect to the line of constant shearing stress so that the regions of reserve of strength slightly exceed the plastic regions. For this given value of cohesion, when the slope is tilted slightly above the critical value, the plastic region encroaches on the region of reserve of strength and the sliding curve finally develops.

III. Theoretical Plasticity Analysis

Part III concerns the development of an approximate mathematical theory for the plastic failure of a soil slope due to its own mass, which would permit the determination of the possible position and form of the sliding curve and of



(a) EXTERNAL BOUNDARY LOADING CONDITIONS. SLOPE OF SAND.

(b) EQUILIBRIUM CONDITIONS

FIG. 10.—HYPOTHETICAL WEDGE OF PLASTIC MATERIAL

the approximate stress conditions leading to failure. A theoretical investigation was made based on the assumption that a wedge is formed by two intersecting planes (OD and Oy in Fig. 10), within which the state of stress has progressed from an elastic to a plastic state.

The general principles of the plasticity theory developed by A. Nádai[11] were used to study the phenomenon of failure of slopes. Fig. 10(*a*) represents the model of a sand slope governed by the following conditions:

(1) A uniform normal external pressure is applied on surfaces AB and AD equal to that creating the equivalent cohesion in the sand model;

(2) Surfaces AB and AD are free from shearing stresses; and

(3) Since the wedge is symmetrical about Oy, the lateral displacement of surface BC is zero.

According to the theory of plasticity the stress in the plastic region is independent of the plastic deformation. The solution of the problem is approximate because it is necessary to assume the entire wedge to be in plastic state, whereas in nature only the region above the rupture surface is in a plastic state.

[11] *Zeitschrift für Physik*, Berlin, 1924, p. 106.

In nature, no rigid boundaries exist which would influence somewhat the stress condition within the plastic zone. In the laboratory, this problem is complicated by the fact that body forces are acting, that the boundary conditions cannot be satisfied readily, and that the solution of nonlinear differential equations is required. Therefore, it was decided to investigate only a certain class of solutions to develop one that would represent the physical phenomenon reasonably well. The theoretical investigation is concerned with determining the plasticity equations, which would define approximately the stress conditions for the plastic state and would permit drawing the potential slip planes in the slope.

In his presentation of the theory of elasticity, S. Timoshenko[12] states the two differential equations of equilibrium required for defining the stresses in the elastic region. For the two-dimensional case, using polar coordinates and the body forces shown in Fig. 10(*b*):

$$\sigma_r + r\,\frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{r\theta}}{\partial \theta} - \sigma_\theta + r\,\gamma \cos\theta = 0 \quad \text{(3a)}$$

and

$$\frac{\partial \sigma_\theta}{\partial \theta} + r\,\frac{\partial \tau_{r\theta}}{\partial r} + 2\,\tau_{r\theta} - r\,\gamma \sin\theta = 0 \quad \text{(3b)}$$

In the plastic region it is necessary to introduce an additional equation, which expresses the fact that, in the plastic state, the stresses are in equilibrium; thus:

$$\left(\frac{\sigma_r + \sigma_\theta}{2}\right) \sin\phi + c\cos\phi = \sqrt{\frac{(\sigma_r - \sigma_\theta)^2}{2} + \tau^2_{r\theta}} \quad \text{(4a)}$$

or

$$\tau_{\max} = \sqrt{\frac{(\sigma_r - \sigma_\theta)^2}{2} + \tau^2_{r\theta}} \quad \text{(4b)}$$

The condition of plasticity may be expressed by either Eq. 4*a* or Eq. 4*b*. Eq. 4*b* represents the common concept of the maximum shearing stress condition of failure; whereas Eq. 4*a* represents the failure condition in a cohesive soil having both an angle of friction, ϕ, and cohesion, c. Eq. 4*b* is used to obtain a solution, and to satisfy the requirement of a constant maximum shear stress throughout the plastic region. It is directly applicable to the problem represented by the gelatin model but only approximately so for a cohesive soil.

A solution of the problem consists in finding a set of functions for the stresses σ_r, σ_θ, and $\tau_{r\theta}$ which satisfies Eqs. 3 and 4*b*, and of selecting values at the boundaries reasonably consistent with the boundary conditions.

The first step is to derive a fundamental equation governing the shearing stress by eliminating σ_r and σ_θ from Eqs. 3 and 4*b*. Writing Eq. 4*b* in the form—

$$(\sigma_r - \sigma_\theta) = \pm\, 2\,\sqrt{\tau^2_{\max} - \tau^2_{r\theta}} \quad \text{(5a)}$$

let the right-hand side (for simplicity) be represented by $\frac{A}{r}$, or:

$$r\,(\sigma_r - \sigma_\theta) = \pm\, A \quad \text{(5b)}$$

[12] "Theory of Elasticity," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York, N. Y., 1934.

Differentiating with respect to r and θ,

$$\pm \frac{\partial^2 A}{\partial r\, \partial \theta} = \frac{\partial^2}{\partial r\, \partial \theta}\left[r\,(\sigma_r - \sigma_\theta)\right] = \frac{\partial}{\partial \theta}\left[(\sigma_r - \sigma_\theta) + r\left(\frac{\partial \sigma_r}{\partial r} - \frac{\partial \sigma_\theta}{\partial r}\right)\right] \quad \text{..(6a)}$$

and

$$\pm \frac{\partial^2 A}{\partial r\, \partial \theta} = \frac{\partial \sigma_r}{\partial r} - \frac{\partial \sigma_\theta}{\partial \theta} + r\,\frac{\partial^2 \sigma_r}{\partial r\, \partial \theta} - r\,\frac{\partial^2 \theta}{\partial r\, \partial \theta} \quad \text{..........(6b)}$$

Differentiating Eq. 3*a* with respect to θ and substituting the value of $\frac{\partial \sigma_\theta}{\partial \theta}$ into Eq. 6*b*:

$$\pm \frac{\partial^2 A}{\partial r\, \partial \theta} = -\frac{\partial^2 \tau_{r\theta}}{\partial \theta^2} - r\,\gamma \sin \theta - r^2\,\frac{\partial^2 \sigma_\theta}{\partial r\, \partial \theta} \quad \text{...........(7a)}$$

Differentiating Eq. 3*b* with respect to r, multiplying through by r, and substituting in Eq. 7*a*:

$$\pm \frac{\partial^2 A}{\partial r\, \partial \theta} = -\frac{\partial^2 \tau_{r\theta}}{\partial \theta^2} + r^2\,\frac{\partial^2 \tau_{r\theta}}{\partial r^2} + 3\,r\,\frac{\partial\, \tau_{r\theta}}{\partial r} + r\,\gamma \sin \theta + 0 - r\,\gamma \sin \theta$$

$$= -\frac{\partial^2 \tau_{r\theta}}{\partial \theta^2} + r^2\,\frac{\partial^2 \tau_{r\theta}}{\partial r^2} + 3\,r\,\frac{\partial \tau_{r\theta}}{\partial r} \quad \text{...............(7b)}$$

The equation in its final form is obtained by substituting Eq. 3*a* in Eq. 7*b*, which then proves to be independent of body forces:

$$\pm \frac{\partial^2}{\partial r\, \partial \theta}\left(2\,r\,\sqrt{\tau^2_{max} - \tau^2_{r\theta}}\right) = -\frac{\partial^2 \tau_{r\theta}}{\partial \theta^2} + r^2\,\frac{\partial^2 \tau_{r\theta}}{\partial r^2} + 3\,r\,\frac{\partial \tau_{r\theta}}{\partial r} \quad \text{.....(8)}$$

Eq. 8 governs the shearing stress at every point in a material in a plastic state at incipient failure. Eq. 8 turns out to be independent of body forces and therefore becomes identical with that obtained by Mr. Nádai[13] in an investigation, which did not involve body forces. Following the general procedure, adopted by Mr. Nádai, solutions of Eqs. 3 and 4*b*, for the plastic state, are investigated herein because they are independent of the coordinate r.

The determination of the stresses in the wedge requires first the solution of the fundamental differential for the stresses σ_θ and σ_r. For this purpose the assumption is made that the shearing stress is a function of θ only. Integrating $\tau_{r\theta}$ with respect to θ:

$$\frac{\partial \tau_{r\theta}}{\partial \theta} = \mp 2\,\sqrt{\tau^2_{max} - \tau^2_{r\theta}} + 2\,C\,\tau_{max} \quad \text{...............(9)}$$

in which the constant of integration is designated by $2\,C\,\tau_{max}$. Integrating Eq. 9, in turn,

$$2\,\theta + \alpha = \int \frac{\partial \tau_{r\theta}}{\tau_{max}\,C \mp \sqrt{\tau^2_{max} - \tau^2_{r\theta}}} \quad \text{.............(10)}$$

[13] "Plasticity," by A. Nádai, Eng. Societies Monographs, McGraw-Hill Book Co., Inc., New York, N. Y., 1931.

An auxiliary function ψ is introduced such that

$$\tau_{\max} \sin \psi = \tau_{r\theta} \quad \text{(11)}$$

Eq. 10 then becomes

$$2\,\theta + \alpha = \int \frac{\cos \psi}{C \mp \cos \psi}\, \partial\, \psi \quad \text{(12)}$$

Using the lower plus sign of Eq. 12 only:

$$\partial \theta = \frac{\cos \psi}{2\,(C + \cos \psi)}\, \partial \psi \quad \text{(13)}$$

Eq. 13 represents the conditions in which the stresses are brought into play in resisting the boundary loading, as distinguished from the case, in which the boundary loads are induced as reactions of the stresses.[12] From the equilibrium Eqs. 3 and 4*b*, by substituting the value of $\frac{\partial \tau_{r\theta}}{\partial \theta}$ and $\tau_{r\theta}$ from Eqs. 9 and 11 and introducing now the body forces:

$$r\,\frac{\partial \sigma_r}{\partial r} = -\,2\,\tau_{\max}\,C - r\,\gamma \cos \theta \quad \text{(14a)}$$

and

$$\frac{\partial \sigma_\theta}{\partial \theta} = -\,2\,\tau_{\max} \sin \psi + r\,\gamma \sin \theta \quad \text{(14b)}$$

Having determined the values of $\partial\theta$ and $\partial\psi$, the fundamental differential stress Eqs. 14 for σ_r and σ_θ can now be solved by substituting for $\partial\theta$ from Eq. 13. Eq. 14*b* then becomes

$$\partial \sigma_\theta = -\int 2\,\tau_{\max}\,\frac{\sin \psi \cos \psi}{2\,(C + \cos \psi)}\,\partial \psi + \int r\,\gamma \sin \theta\, d\theta \quad \text{(15)}$$

Integrating Eq. 15, the expression for σ_θ is:

$$\sigma_\theta = +\,\tau_{\max}\,(C + \cos \psi) - \tau_{\max}\,C \log\,(C + \cos \psi) - r\,\gamma \cos \theta + f_2(r) \quad \text{(16)}$$

in which the constant of integration is a function of r only. Integrating Eq. 14*a*:

$$\sigma_r = -\,2\,\tau_{\max}\,C \log r - r\,\gamma \cos \theta + f_1(\psi) \quad \text{(17)}$$

in which the constant of integration is a function of ψ only. In order to satisfy the condition of plasticity (Eqs. 4*b* and 11), the variables in r and θ must vanish identically. Since—

$$\sigma_r - \sigma_\theta \approx 2\,\tau_{\max} \cos \psi \quad \text{(18)}$$

—it follows that the functions of $f_2(r)$ and $f_1(\psi)$ must equal, respectively,

$$f_2(r) = -\,2\,\tau_{\max}\,C \log r + C_1 \quad \text{(19a)}$$

and

$$f_1(\psi) = \tau_{\max}\,[\cos \psi - C \log\,(C + \cos \psi)] + \tau_{\max}\,C + C_1 \quad \text{(19b)}$$

Let the constant

$$C_1 = \tau_{max}\, C \log a^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20)$$

in which a is a length measured radially from the apex of the wedge. Substituting these values of the functions into Eqs. 16 and 17 the equations for the stresses σ_r and σ_θ in their final form are:

$$\sigma_r = -\tau_{max}\, C \log \frac{r^2}{a^2}(C + \cos\psi) - \tau_{max}\cos\psi - r\,\gamma\cos\theta + \tau_{max}\, C \quad (21a)$$

and

$$\sigma_\theta = -\tau_{max}\, C \log \frac{r^2}{a^2}(C + \cos\psi) + \tau_{max}\cos\psi - r\,\gamma\cos\theta + \tau_{max}\, C \ldots (21b)$$

To obtain the values of the function ψ Eq. 12 is integrated to yield the values of ψ implicitly as a function of θ and C:

$$2\,\theta + \alpha = \pm \frac{2\,C}{\sqrt{C^2 - 1}} \tan^{-1}\left(\frac{\sqrt{C+1}}{\sqrt{C-1}}\tan\psi/2\right) \mp \psi \ldots\ldots\ldots (22)$$

The curves in Fig. 11 were obtained from Eq. 22 (with $C > 1$ and $0 < \theta < \theta_0$) for determining the numerical values of the stress Eqs. 21.

The general expressions for the stresses σ_r, σ_θ, and $\tau_{r\theta}$ (see Eqs. 11 and 21) should satisfy the boundary conditions of the soil slope. These conditions require that the shearing stress $\tau_{r\theta}$ be equal to zero along line OD. (θ equals θ_0, the wedge angle in Fig. 10(a)) and that the normal stress σ_θ be approximately uniform as originally assumed along the boundary OD.

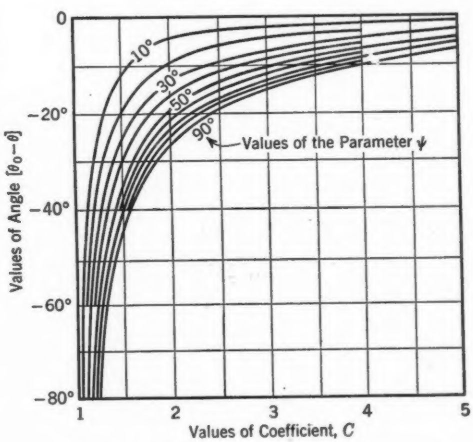


FIG. 11.—VALUES OF THE PARAMETER ψ FOR DETERMINING THE SHEARING STRESS τ IN A WEDGE OF PLASTIC MATERIAL

1. Boundary Conditions.—A study of the curves of Fig. 11 shows that in order that the shearing stress given by Eq. 11 may have real values throughout the entire wedge, it must have its maximum value $\tau_{r\theta} = \tau_{max}$ along boundary OC, Fig. 10, where ψ is equal to 90° by Eq. 11. Along plane OD where θ equals θ_0 the shear stress must be equal to zero. Hence, by Eq. 11, ψ must also equal zero along plane OD. A plastic region exists where τ_{max} has become equal to the maximum shearing strength that can be mobilized.

For a wedge angle θ_0 of 40°, the value of coefficient C is determined from Fig. 11 for $\theta = \theta_0$ and $\psi = 90°$, where $\tau_{r\theta}$ is a maximum, and C is found to be 1.6. The value of τ_{max} = (10 lb per sq ft) was obtained from Fig. 2 for a slope of 50° (θ equals 40°).

A comparison of the magnitude of σ_θ for the boundary OD in Fig. 12 for wedge angles of 40° and 50°, respectively, shows that lower boundary stresses are required for a wedge angle of 50° in a plastic condition (that is, less equivalent cohesion is required for stability) than for a wedge angle of 40°. On the boundary OC the normal stress should increase linearly in the y-direction in accordance with the usual concepts of earth-pressure phenomenon. This condition is satisfied reasonably well except near the top of the slope. The normal stress loading on boundary OD, on the other hand, does not satisfy quite so well the required assumed condition of a uniform loading, except possibly in the upper part of the slope. It is believed that the distribution of σ_θ is reasonably uniform, as assumed, and that it represents a good approximation for the upper sliding curves. The boundary condition for the wedge can therefore be assumed to be satisfied reasonably well. Fig. 12 shows also that, in the upper part of the slope, tension can exist (σ_r is positive); and, therefore, tension cracks may develop in cohesive soil.

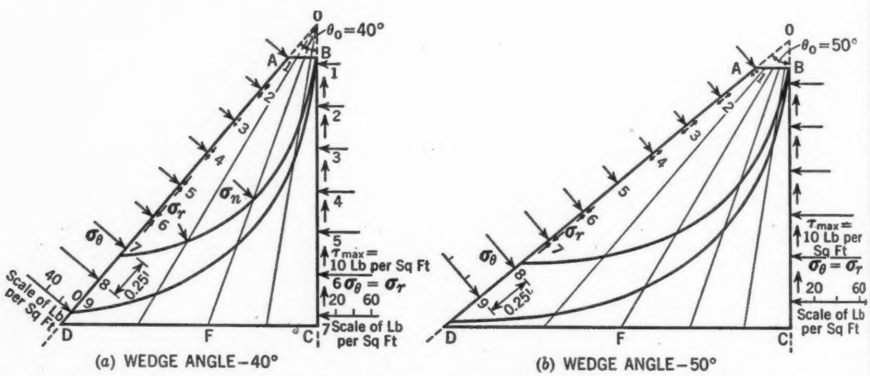


FIG. 12.—DETERMINATION OF SLIP LINES BY THE THEORY OF PLASTICITY, ASSUMING THE BOUNDARY CONDITIONS NECESSARY TO CAUSE FAILURE

2. The Slip Lines.—The angle β that the maximum shearing stress makes with the radial line through a given point $(r_1\theta)$ is given by:

$$\tan (2\,\beta) = \frac{\sigma_r - \sigma_\theta}{\tau_{r\theta}} = \frac{2\,\tau_{\max} \cos \psi}{2\,\tau_{\max} \sin \psi} = \cot \psi \ldots\ldots\ldots\ldots (23)$$

When the directions of maximum shearing stress are found by Eq. 23 and Fig. 11, for a large number of points in the plastic region, the slip lines can be drawn as shown in Fig. 12 for wedge angles of 40° and 50°. These slip lines are arcs of circles and have the same general forms as the sliding curve actually observed in nature and in the model tests.

The stress conditions at failure were investigated along the sliding curve by the photoelastic method (inclination of gelatin slope, 49°), Swedish circular arc method (inclination of sand slope, 49°), and by the theoretical plasticity method (analysis of the potential slip line, wedge angle, and slope inclination, 45°). The distribution of the shearing stress in relation to the shearing strength

along the sliding curve is shown in Fig. 13. The stress and strength relation obtained from the Swedish circular arc method is quite different from that determined by the photoelastic and plasticity analyses, which indicate that failure is initiated in a plastic region near the top of the slope which encroaches on the region having a reserve of strength, until a failure curve develops.

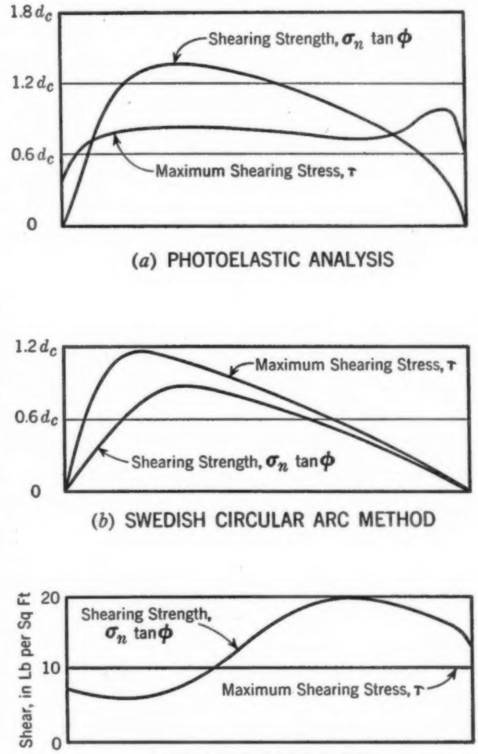


Fig. 13.—Distribution of Maximum Shear Stresses and of the Shearing Strength Along the Slip Lines

Conclusion

The angle of repose of cohesionless soil was found to be entirely a superficial phenomenon. Therefore, it is independent of the height of the slope. The angle of repose for any given material lies between two limiting values—a maximum just before a slide, and a minimum just after a slide. It appears to be independent of the density but is affected by the coarseness of the material. On the other hand, the inclination and the height of a stable slope of a cohesive soil were found to be direct functions of the cohesion and were greatly influenced by the density of the material. By creating an equivalent cohesion in a cohesionless soil, it was possible to reproduce slides in the laboratory which were remarkably similar to those observed in nature, the failure curve receding from the surface deeper into the slope with increasing cohesion. It was also possible to observe the mechanics of failure, and the deformations and strains within the slope. The distribution of strain along the failure curve was found to form a consistent pattern with a maximum near the top of the slope where a failure appeared to be initiated after the strain exceeded a rather well-defined value.

The photoelastic studies in gelatin models afforded a clear picture of the distribution and relative order of magnitude of the maximum shearing stress in the slope. There was a close resemblance between the pattern of the shearing stresses and the pattern of the shearing strains obtained by means of the sand model, the plastic regions coinciding with the regions of maximum strain. The plastic regions develop gradually near the top of the slope and encroach

upon the regions of reserve of strength, until a rupture curve is formed, followed by a slide.

The plasticity theory permitted the determination of the possible position and form of the sliding curves and of the approximate stress conditions leading to failure. The slip lines were found to be arcs of circles which agree closely with observed natural slide phenomena. The analysis shows that it is possible to have tensile stresses in the upper part of the slopes which might lead to tension cracks and final failure. A failure is imminent whenever the height and inclination of slope and the shearing properties of soil bear such a relation to each other that a plastic region develops.

Acknowledgments

This paper is based on a thesis entitled "Stability of Soil Slopes."[14] The writer wishes to express his appreciation to the following members of the faculty of Columbia University in New York, N. Y.: J. K. Finch, M. ASCE, chairman of the Civil Engineering Department and now Dean of the Engineering School; to D. M. Burmister, Assoc. M. ASCE, for advice, encouragement, and innumerable and invaluable discussions on every aspect of the research and critical review of this paper; and to R. D. Mindlin, Assoc. M. ASCE, for his helpful suggestions and advice on the photoelasticity and plasticity analysis. G. P. Tschebotarioff, M. ASCE, of Princeton University at Princeton, N. J., and M. G. Salvadori, Assoc. M. ASCE, of Columbia University, reviewed the paper critically before presentation.

[14] "Stability of Soil Slopes," by Ek-Khoo Tan, thesis submitted to Columbia Univ., New York, N.Y., in 1945, in partial fulfilment of the requirements for the degree of Doctor of Philosophy in Engineering.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

STRENGTH OF THIN STEEL COMPRESSION FLANGES

Discussion

BY BRUCE G. JOHNSTON, AND EDWARD L. BROWN AND DON S. WOLFORD

BRUCE G. JOHNSTON,[28] M. ASCE.[28a]—The tests[29] made at Lehigh University at Bethlehem, Pa., by Lloyd Cheney, Jun. ASCE, and the writer, supplement the results presented in Fig. 14 of the paper by Professor Winter. In the Lehigh University tests, the flanges of both carbon and silicon steel 10WF49 sections, each from a single rolling, were planed to different thicknesses, thereby providing a range of width to thickness ratios with a minimum variation in yield point.

Details of the tests will not be discussed, since they have already been reported,[29] but the results are summarized in Fig. 21.

For a very long plate, with one longitudinal edge simply supported and the other free S. Timoshenko[30] gives a coefficient of 0.46 in the elastic buckling formula, whereas the author quotes a factor of 0.50 in Eq. 9. Professor Timoshenko's coefficient is based on Poisson's ratio of 0.25, corresponding to neither steel nor aluminum, for which coefficients of 0.43 and 0.41 are obtained[31,32] for Poisson's ratios of 0.30 and 0.33, respectively. These differences are somewhat academic, since the buckling strength will be affected by the torsional stiffness of the thick section at the juncture of the web and flange, and by the bending stiffness of the web, which may be negative or positive.

NOTE.—This paper by George Winter was published in February, 1946, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1946, by Fred T. Llewellyn, and Jacob Karol; October, 1946, by Robert L. Lewis and Dwight F. Gunder; and December, 1946, by L. C. Maugh and L. M. Legatski.

[28] Associate Director, Fritz Eng. Laboratory, Lehigh Univ., Bethlehem, Pa.

[28a] Received November 25, 1946.

[29] "Steel Columns of Rolled Wide Flange Section," by Bruce Johnston and Lloyd Cheney, *Publication No. 190*, A.I.S.C., November, 1942.

[30] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1st Ed., 1936, p. 340.

[31] "Theory of Elastic Stability Applied to Structural Design," by Leon S. Moisseiff and Frederick Lienhard, *Transactions*, ASCE, Vol. 106, 1941, p. 1059.

[32] "Chart for Critical Compressive Stress of Flat Rectangular Plates," by H. N. Hill, *Technical Note No. 773*, National Advisory Committee for Aeronautics, August, 1940.

However, if no account is to be taken of these effects, the question arises as to whether a coefficient of 0.43 is not preferable to one of 0.50.

In the tests at Lehigh University, in the case of large values of $\frac{b_w}{t}$ the web was thicker than the planed flanges, providing a partly fixed edge, and yielding

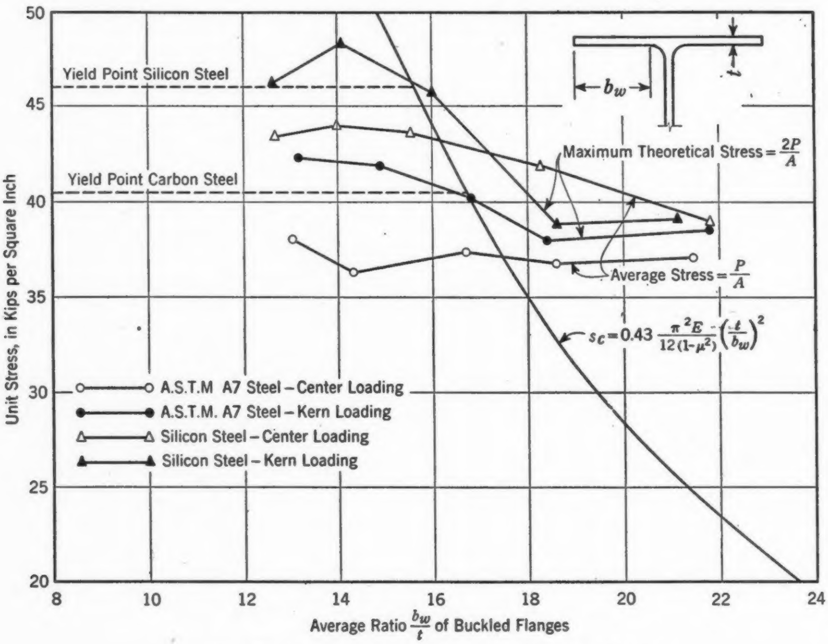


FIG. 21.—STRESS IN BUCKLED FLANGE AT MAXIMUM LOAD

effective coefficients even greater than 0.5; in fact, the buckling in all the tests was plastic rather than elastic. The results presented in Fig. 21 are self-explanatory and show that flanges of rolled shapes may be expected to develop at least 90% of the yield point of the material, within the range of $\frac{b_w}{t}$ in which plastic buckling is predicted by the elastic buckling formula.

EDWARD L. BROWN,[33] ESQ., AND DON S. WOLFORD,[34] ESQ.[34a]—More accurate methods for computing strength and other properties of structural sections formed of thin flat-rolled metals have been needed for some time. Previous methods were mainly deficient because they did not adequately evaluate the strength of compression flanges. Designers have become accustomed to assuming certain maximum multiples of the thickness to be fully effective in stiffened flanges, which has been a fairly successful practice in reasonably

[33] Engr., American Rolling Mill Co., Middletown, Ohio.
[34] Senior Research Engr., American Rolling Mill Co., Middletown, Ohio.
[34a] Received November 29, 1946.

compact sections. However, it was evident that such simple relations did not adequately define behavior for all cases. Professor Winter presents relations in his paper by which such flanges may be evaluated, taking both b/t-ratio and stress level into account. Sections discussed in this paper cover flanges with b/t-ratios up to 170. The purpose of this discussion is to present test data made on sections containing compression flanges with b/t-ratios ranging from 242 to 429.

These sections were formed of 18-gage and 20-gage mild steel. All were 3 in. deep with compression flanges of 12-in. and 16-in. widths. They were similar to the U-beam in Fig. 3, except that the lower flanges were lipped and one flange was turned outward to provide a joint for adjacent sections. These tests were sponsored jointly by the American Iron and Steel Institute and Cornell University at Ithaca, N. Y., and were made under Professor Winter's supervision. They were witnessed by the writers of this discussion who also made the calculations and analysis of the results.

Four identical pairs of each type of section were subjected to quarter-point beam loading using an 80-in. span. Deflections and strains in top and bottom fibers were observed as load was applied and released at successively higher values. The yield-point load was assumed to be that which first caused a large increase in residual deflection upon release of load.

Section properties were computed for each type of section using (1) gross section and (2) effective section. The effective widths were determined from curves computed by Eq. 6, carried out to b/t-ratios in the range needed. The measured tensile yield strengths of the steels used were taken into account. Computed and observed deflections and loads are compared in Table 6.

TABLE 6.—COMPARISON OF ACTUAL AND COMPUTED VALUES

Type	Dimensions, in Inches			Deflection,[b] in Inches			Load, in Pounds[c]		
	Width	Thickness	Ratio[a] b/t	Observed in test	Computed: Effective width	Computed: Gross section	Observed in test	Computed: Effective width	Computed: Gross section
320.....	12	0.037	321	0.25	0.30	0.16	1,267	1,212	1,912
328.....	12	0.049	242	0.25	0.28	0.17	1,725	1,610	2,070
360.....	16	0.037	429	0.24	0.30	0.15	1,350	1,179	1,880
368.....	16	0.049	323	0.28	0.28	0.16	1,725	1,683	2,168
Average ratio, $\frac{\text{computed value}}{\text{test value}}$				...	1.14	0.63	...	0.935	1.340

[a] The ratio of width (between webs) to thickness. [b] Deflections are at working stress level, assumed as the yield point divided by 1.85. [c] Loads are for a single section and are at the yield point.

Computed loads based on effective-width properties averaged 6.5% lower than observed loads, indicating good correlation tantamount to safe and efficient design. Gross section properties led to expected loads that were higher than actually obtained, and were therefore misleading.

Deflections shown in Table 6 are at working stress levels, determined by dividing the yield point by 1.85. It is the deflection at this stress level that

is pertinent in design. The moment of inertia used to determine deflection is based on the effective width of the compression flange at the working stress level. A larger part of the flange is effective at this level than is effective at the yield-point stress level.

The calculated deflections averaged 14% greater than those observed in the test, which is reasonable. Deflections calculated by gross section properties were only 63% of those observed in test, which shows that such an approach is not reliable.

Top and bottom strains were approximately equal at the yield loads, placing the neutral axes near mid-depth where computations using properties based on effective widths indicated they should be, rather than near the compression flange as indicated by gross properties.

In summary, these additional tests and correlations show definitely that the relations given by Professor Winter for determining the effective widths of compression flanges, stiffened along both edges, enable the designer to compute design loads and deflections quite accurately at b/t-values at least as high as 429.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

FUTURE COSTS AND THEIR EFFECTS ON ENGINEERING BUDGETS

Discussion

BY ISADOR W. MENDELSOHN

ISADOR W. MENDELSOHN,[2] M. ASCE.[2a]—The subject of this timely paper concerns every engineer intimately. Construction costs are rising and are in a stage of unpredictable flux. As to the phenomena responsible for increasing construction costs, it is necessary to study the entire construction industry critically and to evaluate the numerous factors and their interplay. Anything short of such analysis will provide only an unbalanced judgment. When, in four of his seven conclusions, Mr. Howson stresses the relationship of wage rates to high construction costs and in none of the conclusions specifies definitely any other influences contributing to such high costs, it is a moot question as to whether his analysis of factors of construction costs is complete.

Characteristics of the Construction Industry.—The construction program in the United States for the period from 1915 to 1944 comprises new, work relief, and maintenance construction. In past years the cost of new work has varied from $2,350,000,000 in 1933 to $13,498,000,000 in 1942,[3] averaging $6,700,000,000 annually. Maintenance construction has averaged $3,000,000,000 annually, whereas work relief construction for the period from 1933 to 1943 has averaged $636,000,000. New construction includes private and public residential and nonresidential buildings, farms, public utilities, military and naval work, pipe lines, highways, sewage disposal, water supply, conservation, and development. In hearings before the Temporary National Economic Committee on Investigation of Concentration of Economic Power, Isador Lubin testified[4] that the construction industry employed 5.5% of nonagricultural workers in 1929 and consumed about 15% of the commodities produced in the United States for the period from 1919 to 1935. Willard Thorp stated[4] the construction in-

NOTE.—This paper by Louis R. Howson was published in March, 1946, *Proceedings*.

[2] San. Engr., Washington, D. C.

[2a] Received August 27, 1946.

[3] "Total Construction Activity in the Continental United States," Constr. Statistics Section, Constr. Div., Bureau of Foreign and Domestic Commerce, U. S. Dept. of Commerce, Washington, D. C., January, 1946.

[4] "Construction Industry," Temporary National Economic Committee, Pt. 11 of "Investigation of Concentration of Economic Power," June 27–July 14, 1939 (*No. Y4.T.24:Ec. 7/Pt. 11*, Supt. of Public Documents, Washington, D. C.).

dustry has certain definite characteristics: (1) It produces a product which is not uniform in type, design, size, and location, and operates on a made-to-order basis; (2) the products are far from simple, requiring thousands of materials and a variety of skills; (3) the demand is irregular, essentially local, and highly seasonal. Considering the third point, Beardsley Ruml has aptly stated that the circumstances which result in a profitable outlook for the building of a factory by one company will simultaneously affect thousands of companies.[5]

According to the Social Security Board, there were, in 1938, 22,000 general building contractors, 8,000 other general contractors, and nearly 67,000 special trade contractors, involving a total of more than 825,000 employees. To these should be added about 50,000[4] who operate "on their own" in the construction industry.

In comparison with other industries, Mr. Ruml finds that the construction industry is costly and wasteful.[5] Pertinent factors[4] are: (1) Difficulty of organizing and planning an efficient work program, bringing many parts together in varying ways, places, and on various types of operations; (2) expensive purchasing, in small lots; (3) expensive selling practices; (4) uneven flow of work and employment; (5) little research, and resistance to new methods and materials; (6) inadequate and unstable financing; (7) bitter competitive practices leading to agreements, collusions, price controls, use of building codes, and union restrictions; (8) a tendency for each part of the construction industry to increase its own charges rather than to reduce them; (9) high transportation costs of construction materials (in 1936, 57% of the value of gravel and sand at destination, 28% of that of common brick, and 21% of that of lumber, shingles, and laths, being absorbed by freight charges); and (10) high degree of concentration of construction material production, showing relatively few producers for many important items, four leading companies[4] cutting 22.8% of the Douglas fir; 7% of the southern pine; and 49.3% of the structural steel shapes.

According to Mr. Ruml:[5]

> "Architects, building contractors, building supply companies, labor, finance and mortgage companies, all found it necessary—each in its own way—to establish and to hold a price structure high enough so that the days and hours of activity would pay for the time when there was little or nothing to do."

Unreasonable Restraints of Trade in the Construction Industry.—In the opinion of Thurman Arnold,[4] unreasonable restraints of trade are the most conspicuous reasons for high construction costs. Restraint practices developed in government proceedings involve producers and distributors of building materials, contractors, and labor. Producers of building materials have fixed prices by private arrangement.

Distributors of building materials try to raise the price of their services by establishing a fixed markup between the price they pay the manufacturer and

[5] "Public Works and Construction After the War," by Beardsley Ruml, address before New York Building Cong., New York, N. Y., April 24, 1945.

the price at which they sell. Many groups of contractors set up little closed markets from which they exclude outside contractors or new types of services.

The building-trades unions have frequently been used as the "strong-arm squads" for collusive agreements among contractors, refusing to supply labor where the contractors' ring wishes labor withheld. In other cases the unions themselves have refused to permit the use of new products or new processes.

Restraints of trade in building have: (*a*) Harassed, boycotted, and eliminated competitors able and willing to reduce prices; (*b*) handicapped the use of prefabricated materials and thwarted the development of methods of mass production in the industry; and (*c*) prevented experiment in housing design, materials, and methods of construction.[4]

United States Department of Justice Action Against Restraints of Trade.—Since the passage of the Clayton Act in 1914, the Department of Justice has instituted approximately 674 cases,[6] of which 160 have involved some parts of the construction industry. In 1939 the Anti-Trust Division of the Department launched a broad attack on restraints in the building industry. Preliminary surveys were made in twenty-six cities. Grand juries were assembled in eleven cities. By September, 1940, ninety-eight indictments involving 1,281 defendants had been returned. Among the defendants were associations of manufacturers, distributors, contractors, and labor; and individual officers of trade associations, business enterprises, and labor unions. In addition to the criminal cases, nineteen equity proceedings were instituted, involving a total of 367 defendants (as stated by Thurman Arnold in an unpublished address before the Illinois Association of Real Estate Boards at Chicago, Ill., on September 20, 1940).

From this campaign economic results appeared more quickly than did legal results. In May, 1939, the Pittsburgh city engineer[7] drew up an estimate of the cost of the electrical work in a new municipal hospital being built with funds of the Public Works Administration (P.W.A.). His estimate was $105,000; the city advertised for bids, opened them, and found that the lowest was $154,000. Specifications were revised and the city re-advertised. The lowest bid was then $148,000, which was rejected. The third set of bids brought a low offer of $152,000.

About that time a team of eight men from the United States Department of Justice reached Pittsburgh and began its investigation. This team advised rejection of the latest bids. On November 3, a federal jury indicted twelve electrical contractors, a trade association, and forty-five individuals, charging a conspiracy to defraud through collusive bidding. A few days later the city received a new set of bids for this hospital electrical work—with a low, this time, of $117,000.

A little later the city[7] opened bids for the purchase of sand and gravel. For the first time in years, the sand and gravel bids received were not identical, and the quoted prices dropped from the prevailing level of $2.25 a ton to from

[6] "Anti-Trust Cases in the Construction Industry," U. S. Dept. of Justice, Washington, D. C., July 1, 1946.

[7] "War and Prices," Temporary National Economic Committee, Pt. 21 of "Investigation of Concentration of Economic Power," December 4–8, 1939 (*No. Y4.T.24:Ec. 7/Pt. 21*, Supt. of Public Documents, Washington, D. C.).

$1.65 to $1.80 a ton—a saving of $17,000 on sand and gravel for the first quarter of 1940.

In another case a low-cost housing project in Pittsburgh was planned in two units, bids on the first being opened before the Anti-Trust Division began investigating and those on the second after the federal grand jury had been sitting about three months. The investigation resulted in bids lowering the electrical contract by 23.5%, the heating contract 27%, the plumbing contract 26%, and the general contract 12%. The over-all reduction in the cost per family on the second unit was $148 per room, or 17% of the total cost.

A federal grand jury in New Orleans returned an indictment on February 21, 1940, against three organizations representing lumber interests. These groups were charged with conspiracy to fix prices, to curtail output, to enforce an agreed policy of distribution, and to restrain trade by abuse of grading, grade marking, and inspection. All these practices were subsequently enjoined.

Federal Trade Commission (F.T.C.) Action Against Restraints of Trade.—Illustrative of many unlawful monopolistic cases in restraint of trade concerning construction activity processed by the F.T.C. are the following:

1. Approximately thirty-one corporations engaged in the business of manufacturing water gate valves, hydrants, fittings, and similar products,[8] and in the sale of such products to cities and state and federal governments, comprised substantially all the manufacturers of such products used for water supply systems. They were located in seventeen different states. The monopolistic practices in this case consisted of agreements by these corporations to fix and maintain prices. The commission found that these practices had actually hindered price competition and had created in the members of this group a monopoly unlawfully restraining interstate commerce.

2. In 1937[8] the commission issued findings and an order against an organization of building material dealers with a membership of more than 150. The materials in which the organization dealt included cement, brick, tile, clay products, sewer pipe, sand, gravel, stone, lime, lumber, roofing, and other materials ordinarily used in the construction industry. The main objective of these associations and their members was to confine retail distribution in building materials and supplies to recognized dealers and to prevent the direct sale by manufacturers to all others.

In 1935 the United States, through the Procurement Division for the Relief Administration,[8] sent to manufacturers in one state an invitation for bids on 100,000 bbl of cement. As a result of action by the state association of builders' supplies, no cement company would quote prices. When the same invitation was mailed to manufacturers outside of that state, again no direct bids were received.

The commission found that interstate commerce in the sale and distribution of building materials was thus restrained because the organized collusion eliminated the so-called irregular dealers and manufacturers and producers

[8] "Federal Trade Commission Report on Monopolistic Practices in Industries," Temporary National Economic Committee, Pt. 5-A of "Investigation of Concentration of Economic Power," March 2, 1939 (*No. Y4.T.24:Ec. 7/Pt. 5-A*, Supt. of Public Documents, Washington, D. C.).

selling to such dealers. Competition in the sale of all building materials was substantially lessened. Competitors of members were unable to obtain interstate shipments of their requirements. Costs to the consuming public were increased.

Identical Delivered Price Systems in the Construction Industry.—A paramount economic factor in unfair pricing in the construction industry is the identical delivered price systems, such as the basing-point, the freight-equalization, and the zone systems. According to the Procurement Division of the United States Treasury Department, during the period from December, 1937, to November, 1938, 76,000 bid openings, or 24.1% of 332,000 bid openings during that period, involved the receipt of identical bids from different sellers. This covered industries including all the major categories from which the government makes purchases.[9]

Concerning the basing-point system,[8] the federal government was in need of steel sheet piling for public jobs at Morehead City, N. C., at Miami, Fla., and for the Triborough Bridge in New York, N. Y., in 1933. The government received bids from only the four companies in the United States making this sheet piling, and those bids were alike for each one of the three jobs delivered at the three sites. The bids were determined by the basing-point system—for example, on the Miami project, freight from Pittsburgh to Miami plus the base price of the Pittsburgh manufacturer. Whichever one of these freight-plus-base price arrangements to Miami was lowest—that was the price charged by all four companies.

The price was identical for the unwelded piling per 100 lb, for piling corners, piling T-sections, fabrication, the extra in welding in pairs, copper content, and all-rail freight factor.

The president of a large corporation testified[8] regarding steel:

> "We generally make the prices.* * * If anyone in the industry makes a lower base price, then that price is taken * * * and becomes a part of the basing-point formula, and all delivered prices are identical again."

Among the industries[8] with basing-point systems are basic industries—steel, pig iron, cement, and lumber—and other industries—builders' supplies, road machinery, vitrified clay sewer pipe, construction machinery, reinforcing materials, valves and fittings, and cast-iron pressure pipe.

Identical delivered prices undermine incentives to economical production and distribution and enable obsolete mills to continue long after they would have gone out of business if price competition had prevailed. Above all, they deprive the public of the benefit of price competition.

In testimony on the basing-point procedure, Harold L. Ickes stated that[8] in purchases by the federal government between June, 1935, and March, 1936, under his administration as Secretary of the Interior, identical bids occurred at least 257 times involving a gross expenditure of $2,866,252.97. Identical bids were received for structural steel, steel tanks, valve boxes, well drilling, fire hydrants, pumps, plumbing and heating specialties, aerators (sewage), cast-iron pipe, water meters, and filter equipment.

[9] "Cartels," Temporary National Economic Committee, Pt. 25 of "Investigation of Concentration of Economic Power," January 15–19, 1940, p. 13,330 (*No. Y4.T.24:Ec. 7/Pt. 25*, Supt. of Public Documents, Washington, D. C.).

Building Codes.—Another of the numerous aspects of the construction industry are the building codes. Such codes[10] are important and necessary in construction, primarily to protect safety and health and next to allow for all the efficiency that industry and enterprise can provide. A survey of the National Bureau of Standards indicates that 25% of the codes have not had a thorough overhauling for more than 20 years, whereas another 23% have not had one for from 16 to 20 years. An extensive survey[11] of building legislation by the John B. Pierce Foundation indicates that a complete revision of the Chicago code is desirable because of its obsolescence, restrictiveness of requirements, and too much variation from generally recognized material standards.

Wide divergence of building code requirements among cities increases costs. Allowable working stresses in concrete, for instance, vary from 500 lb per sq in. to 1,000 lb per sq in.; minimum thicknesses of brick basement walls vary from 8 in. to 16 in. for the same height and load; live-load requirements for dwellings vary from 25 lb per sq ft to 80 lb per sq ft; minimum permitted floor areas for the same type of room vary from 60 sq ft to 120 sq ft; and minimum ceiling heights vary from 7 ft to 9 ft.

Analysis of a random selection of codes showed that the city requiring the lowest amount of pipe (by size and weight) in a one-story house saved 30% of the total amount of cast iron required by the city with maximum requirements. The costs under the minimum code were 80% of the costs under the maximum code for metals other than cast iron[10]—a saving of 20%.

Construction Cost Index, Engineering News-Record (E.N.-R.)—It is noted that the E.N.-R. construction cost index is referred to especially by Mr. Howson in developing his thesis of close relationship between wage rates and construction costs. It has been stated[11] that the E.N.-R. construction cost index:

> "* * * measures wage rate and material price trends. It is not adjusted for labor efficiency, competitive conditions, management, mechanization or other intangibles affecting construction costs."

—Nor does it reflect quality of materials. It is well to note that none of the indexes listed[11] by the E.N.-R. construction cost index contains most of the mentioned factors and but one includes productivity of labor and efficiency of plant and management.

According to a letter dated June 28, 1946, the components for the original E.N.-R. construction cost index were chosen by selecting an imaginary cube of construction that would be composed of 2,500 lb of structural steel, 6 bbl of cement, 600 fbm of 12-in. by 12-in. long-leaf yellow pine, and 200 hours of common labor. This formula produced a value of 100 for 1913. Now the index has four components[12]—(1) structural steel shapes, Pittsburgh base price, multiplied by 25 cwt; (2) cement price at Chicago, exclusive of bags, multiplied by 6 bbl; (3) lumber (which until 1935 was 12-in. by 12-in. long-leaf yellow pine wholesale at New York and since 1935 2-in. by 4-in. S4S pine and fir in

[10] "Your Building Code," by Miles L. Colean, National Committee on Housing, New York, N. Y., February, 1946.

[11] *Engineering News-Record*, April 18, 1946, pp. 84–87, 108, and 113.

[12] "Survey of Current Business," U. S. Dept. of Commerce, 1942 Supplement, p. 183.

carload lots), E.N.-R. twenty-city average price, multiplied by 1,088 fbm; and (4) common labor, E.N.-R. twenty-city average of wage rates in force, multiplied by 200 hours. Analysis indicates that, as a reliable measure of construction cost trends (magnitude involved or competitive prices), the four components are representative neither of construction materials nor of labor.

Before 1921, the labor component represented less than 50% of the total material and labor weight,[11] whereas since 1921 with the exception of 1923, the labor component has been more than 50% and is now (1946) approximately 60% (see Table 1). Actually, labor on nonbuilding construction (see Table 2) approximates from 20% to 45% of labor and material costs.[13] According to a tabulation of labor and material estimates for various types of federal construction projects during 1945 prepared by the Bureau of Labor Statistics (B.L.S.),[11] the percentage of on-site labor varied from 40.0% to 47.6% for building and nonbuilding construction. Furthermore (see Table 3), common labor on nonbuilding construction approximates 40% of the total labor used (data supplied to the writer by Edward M. Gordon from the records of the B.L.S.). The common labor component of the E.N.-R. construction cost index does not truly reflect the skilled, semiskilled, and supervisory labor comprising the larger part of such construction.

TABLE 1.—FOUR-COMPONENT PERCENTAGES OF E.N.-R. CONSTRUCTION COST INDEX (1913–1945)

Year	Index	Weighted Percentages of Construction Cost Index			
		Labor	Cement	Lumber	Steel
1913.....	100	38.0	7.1	17.4	37.5
1917.....	181.24	31.0	5.8	13.2	50.0
1919.....	198.42	47.0	8.0	13.3	31.7
1921.....	201.81	53.6	8.4	13.2	24.8
1929.....	207.02	52.8	6.5	17.6	23.1
1932.....	156.97	54.4	6.2	14.6	24.8
1935.....	196.44	53.7	6.6	16.8	22.9
1939.....	235.51	58.0	5.6	14.1	22.3
1940.....	241.96	57.7	5.5	15.1	21.7
1945.....	307.74	59.2	4.8	18.9	17.1

Table 2 indicates that the three material components of the E.N.-R. construction cost index represent generally minor portions of the total materials incorporated in the projects.

"Structural Steel Shapes" Component.—The classification, "structural steel shapes," is not representative, in quantity and price, of various forms of steel used in nonbuilding construction. From an extensive study[14] of consumers' prices of steel products made by the B.L.S. in 1943 for the period from January, 1939, to April, 1942, the following conclusions are presented:

a. Actual delivered prices paid by steel consumers deviate frequently from published delivered prices. By published price is meant the sum of published base price at the basing point nearest the consumer, plus published extras applicable to the given specification, plus rail freight from the basing point to the consumer's plant.

b. The relative importance of the three components of the delivered price is approximately: Base price, 79%; extras, 14%; and freight, 7%.

[13] "Construction Studies of Public Works Administration Projects," by George D. Babcock, Director, Eng. Management, Federal Works Agency, Washington, D. C., 1943.

[14] "Labor Department Examines Consumers' Prices of Steel Products," *Iron Age*, April 25, 1946.

c. Base prices alone are neither good measures of the level of prices of steel nor adequate indicators of the comparative prices of different steel products.

d. The delivered price of steel was increased as much as 50% in some instances, because of higher freight costs resulting from changes in basing points.

TABLE 2.—PERCENTAGES OF MATERIAL, LABOR, AND OTHER COSTS FOR GROUPS OF PUBLIC WORKS ADMINISTRATION PROJECTS (1934–1939)

Item	Type of project	Percentage of Construction Cost (100%)			Weighted Average, Construction Projects		Percentage of Total Material Cost				Range of Costs (Thousands of Dollars)	
		Material	Labor	Other	Date	No.	Cement	Lumber	Steel bars	Structural steel shapes	From	To
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	Water Works:											
1	Surface supply	59.0	23.9	17.1	October, 1935	8	5.8	5.2	4.3	6.6	51	610
2	Distribution only	67.0	17.8	15.2	January, 1936	8	0.5	0.2	0.1	3.5	43	268
	Power Plants:											
3	Steam and diesel	75.0	14.1	10.9	October, 1936	8	2.2	1.6	0.6	4.3	71	661
	Dams:											
4	Concrete	48.0	24.8	27.2	August, 1937	2	36.9	8.2	0.6	0.2	491	543
	Electric Power:											
5	Transmission	74.0	18.4	7.6	December, 1936	3	...	19.7	...	...	108	300
6	Distribution	72.2	14.2	13.6	May, 1938	6	...	17.2	...	...	65	256
	Bridges:											
7	Truss and girder	61.6	23.7	14.7	November, 1936	6	10.3	4.0	2.8	62.6	549	2,089
	Sewage Disposal:											
8	Activated sludge	62.6	24.0	13.4	July, 1936	6	12.2	7.4	7.8	2.9	95	334
9	Primary treatment	57.9	30.2	11.9	November, 1936	8	8.2	6.1	6.7	6.6	77	402
10	Trickling filters	62.5	20.0	17.5	December, 1936	6	9.6	5.4	3.2	2.0	46	379
	Sewers:											
11	Sanitary	44.0	36.1	19.9	October, 1936	10	4.8	2.5	...	...	57	498
12	Storm	49.7	26.4	23.9	June, 1935	5	25.2	14.9	3.3	0.1	72	308
	Pavements:											
13	Bituminous	58.4	24.2	17.4	October, 1935	7	8.1	...	0.1	...	32	90
14	Concrete	59.5	22.5	18.0	November, 1935	5	42.7	...	9.1	...	23	110
	Hospitals (Fireproof):											
15	Concrete frame	59.6	28.9	11.5	August, 1935	6	5.3	10.1	4.8	2.8	75	190
16	Steel frame	65.2	26.5	8.3	April, 1936	4	3.4	5.6	1.9	8.4	133	1,200
	Schools (Fireproof):											
17	Concrete frame	56.2	33.0	10.8	January, 1937	7	8.1	12.3	6.7	6.9	94	220
18	Steel frame	59.1	31.4	9.5	June, 1936	5	4.5	10.3	2.8	9.3	89	1,700

TABLE 3.—PERCENTAGES OF VARIOUS CLASSES OF LABOR ON FEDERAL PROJECTS IN 1940

Type	Superintendent	Foreman	Clerks	Skilled	Semiskilled	Common	Other Unskilled
Highway [a]	1.9	8.8	1.5	15.5	32.0	39.6	0.7
Sewer and water [a]	3.6	9.8	1.5	28.5	14.9	40.2	1.5
Civil aviation [a]	1.8	7.6	2.0	24.1	25.1	38.5	0.9
Sanitary sewers [b]		22.3 [c]		35.9 [d]		41.8 [e]	

[a] Two hundred Public Works Administration and other federal projects. [b] Three projects at Greece, N. Y., totaling $327,000. [c] Overhead—that is, superintendents, foremen, and clerks. [d] Skilled and semiskilled labor. [e] Common and other classes of unskilled labor.

e. Actual prices varied from 50% to 135% of the April, 1942, published delivered prices during the period covered, whereas published prices remained stable.

f. For many products a very small proportion of purchases is made at the base price.

g. Extras applicable to structural shapes include size and section, special cutting, length, United States Navy and American Society for Testing Materials specifications, cambering, chemical requirement, milling, splitting beams, painting, special marking, protected shipment, surface finish, federal specifications, quantity, test requirement, and restricted physical test.

Lumber Component.—Lumber used as one of the four components of the E.N.-R. construction cost index does not portray competitive price trends. According to the Federal Trade Commission (F.T.C.),[15] of the total lumber cut in the United States (24,975,000,000 fbm) in 1939, nearly 83% was produced in four Pacific states and eleven southern states. About 58% of the total cut in 1935 was for building construction. A statement of the Interstate Commerce Commission for 1939 shows that out of each dollar at destination the cost of transportation amounted to 20.16¢ for lumber, shingles, and laths and 27.79¢ for cement.

In 1940 the B.L.S. index of lumber prices rose steadily from August 3 to September 7, for a total of more than six points. There was no shortage of lumber, actual or prospective.

The rise in lumber prices was used to justify an increase in the price of low-cost houses of more than 7%. Complaints reaching the Department of Justice suggested that some of these price increases were due to collusive price fixing and restriction of output. Accordingly, federal grand juries investigated the production and distribution of construction lumber. In February and April, 1941, consent decrees were entered in two cases which enjoined lumber associations against restraint in trade practices.[6]

According to the E.N.-R.,[16] current prices for 2-in. by 4-in. lumber delivered in the New York area have reached a new high of from $85 to $95 for southern pine and from $91 to $121 for Douglas fir. The price structure per 1,000 fbm of Douglas fir consists of: Mill price, $37; freight to New York, $25; dealer handling, $5; 30% markup, $20; and a truck delivery charge of from $3.55 to $32.

Another news announcement in the E.N.-R.[17] states that, in federal courts in California, Washington, Oregon, and Arizona, the Office of Price Administration (O.P.A.) filed forty suits against lumber firms, seeking a total of about $9,000,000 in treble damages on charges of illegal activities involved in handling 65,000,000 ft of lumber. The O.P.A. charged that shippers had diverted lumber by sending it to themselves at dummy addresses and holding it there for bargaining; and that mills were producing out-size timbers in order to charge premium prices.

Another aspect of the lumber trade that deserves attention is quality of product delivered. The E.N.-R.[18] carries the news item that the St. Louis (Mo.) area is being flooded with black market lumber, some of it so green that the "* * *

[15] "Report of the Federal Trade Commission on Distribution Methods and Costs," Pt. III, "Building Materials—Lumber, Paints and Varnishes and Portland Cement," F.T.C., Washington, D. C., 1944.

[16] *Engineering News-Record*, June 27, 1946, p. 51.

[17] *Ibid.*, July 4, 1946, p. 1.

[18] *Ibid.*, June 6, 1946, p. 1.

sap practically is running out of it * * *." As one O.P.A. official stated:

> "The great tragedy of this 'racket' is not so much the inflated cost of this housing material, but the loss of investment that will come when this green wood starts to buckle * * *."

Cement Component.—There were 154 active cement manufacturing mills in the United States in 1941 with an estimated annual capacity of about 247,000,000 bbl. About 55% of the total United States Portland cement producing capacity is concentrated in ten companies.[15]

Since about 1916, the delivered prices of cement have been identical at any particular destination in the United States.[6] For example, the United States Navy Department opened bids for cement on November 17, 1943, for eighteen delivery points along the seaboard from Portland, Me., to New Orleans, La. At Boston, Mass., sixteen companies bid an identical price of $2.52 per bbl. At Philadelphia, Pa., fifteen companies bid identically at $1.98 per bbl. The price in Washington, D. C., for sixteen companies was $2.11.

The F.T.C. has had a pending case against the basing-point system in the cement industry for a number of years and the Department of Justice brought a suit against a leading trade organization with about ninety member cement companies in June, 1945. The F.T.C. reported to the United States Senate in 1933 that, through the use of the basing-point system, many commercial and private purchasers have been forced to pay higher prices for cement. The basing-point system has encouraged crosshauling with resultant aggregate increases in freight.[8] On the basis of available data, the F.T.C. estimated that in 1927 there was an average unnecessary burden of 24.3¢ per bbl which, applied to the entire production of that year, made a total of about $42,000,000.

Labor Productivity.—Although the B.L.S. has been making studies on labor productivity since before 1898, it has not as yet developed an adequate measure based upon fundamental concepts in construction activity. Of productivity of labor in industry, W. D. Evans of B.L.S. has stated that,[19] in the period between the wars, most industries achieved large increases in man-hour output. Better processes, machines, handling of materials, organization of jobs—all contributed to improvement of productive efficiency. During World War II, unit labor requirements for liberty ship construction decreased almost 60% from the date the first ship was delivered in December, 1941, to December, 1943.[20] The decline in unit labor requirements reflects the economies of mass-production methods.

Apropos of labor productivity in building construction are some news items that appeared in 1946.[21,22] An Ohio contractor, charged by the Wage Stabilization Board with paying workmen illegal (too high) wages in constructing homes for war veterans, told how he "trimmed" construction costs so that he was able to sell five-room homes with an O.P.A. ceiling price of $7,250 for $5,850 and make money. By using his own superintendents and the best men

---

[19] "Recent Productivity Trends and Their Implications," by W. D. Evans, B.L.S., Washington, D. C., January 25, 1946.

[20] "Productivity Trends in American Industry," B.L.S., Washington, D. C., January, 1946.

[21] *Engineering News-Record*, May 23, 1946, p. 3.

[22] *Ibid.*, July 4, 1946, p. 2.

obtainable, together with labor-saving equipment, and eliminating all subcontractors, this contractor reduced the cost of excavation from $75 to $14 per house; reduced the brickwork cost from $325 to $130 per house; sewer digging and pipe laying costs $100; cement basement floors and steps costs from $220 to $90; rough-in carpentry costs from $300 to $120; plumbing costs from $650 to $350; painting costs $250; electrical work from $140 to $60; and grading costs from $200 to $20 per unit.

Efficiency of Management in Construction Activity.—Lives there an engineer who in the course of his professional practice has not witnessed examples of construction abandoned after little or no use which could have been forestalled by proper planning and management?

In the initial stages of P.W.A. several water distribution systems were installed before suitable water supplies were developed. Some of these supplies did not prove adequate. In the case of a sewage treatment plant, a garbage grinding unit was built but was only tested and has never been operated, because the city street department (which collects the garbage and is not under the authority of the sewage treatment plant) never changed the routes of the garbage collection and now disposes of the garbage by sanitary fill.

According to the E.N.-R.[23] an agreement was reached in 1946 between the District of Columbia and the Washington Suburban Sanitary Commission whereby the sewage from Montgomery and Prince Georges counties, Maryland, will be discharged into the Washington sewerage system for treatment at the District of Columbia plant at Blue Plains. It was learned that the Maryland sanitary commission will abandon the use of its 7.5-mgd primary treatment plant when the connecting sewers are built. This plant, costing about $800,000, was first put in operation in November, 1940.

In the opinion of Miles L. Colean[24] the excessive street and sanitary improvements in the outlying areas of many large cities during the economic boom of the 1920's are unwarranted public improvements due to political pressure.

Even in these days of high construction costs, low prices can be obtained by good planning. The previously mentioned case of the Ohio contractor is one example. In another, a low unit cost of $2.09[25] per sq ft was achieved for a one-story frame factory building of 27,000 sq ft erected in Los Angeles County, California, as a result of attention given to economy in preliminary studies of five schemes considered by the consulting engineers and architects.

As to the dependability of E.N.-R. construction cost index for measuring labor and material price trends, it is noted Mr. Howson reported elsewhere,[26] in 1946, that it is practically the universal experience of those attempting to let work by contract that the prices bid exceed greatly those estimated by adjusting prior construction costs according to the construction cost indexes, including E.N.-R.

---

[23] "District Will Treat Maryland Sewage," *Engineering News-Record*, June 27, 1946, p. 54.

[24] "Stabilizing the Construction Industry," by Miles L. Colean, National Planning Assn., February, 1945.

[25] "Cost Study of One-Story Factory," *Engineering News-Record*, April 18, 1946, pp. 102–103.

[26] "Construction Costs and Water Rates," by Louis R. Howson, *Journal*, A.W.W.A., June, 1946, p. 747.

Furthermore, E.N.-R. also considers that[27] uncertainties about material and labor supply, spotty conditions on labor efficiencies, and low productivity on some jobs provide the "invisible" costs that push many bids up higher than would be indicated by the cost indexes that measure material prices and labor wage trends.

In his paper Mr. Howson states that the steep rise of the E.N.-R. construction cost index from 1932 to 1940 is a reflection of many factors, including general deterioration in labor output and efficiency. In this view Mr. Howson seems to be at variance with the E.N.-R. which states[28] that the weighted indexes (including E.N.-R. construction cost index) measure only hourly wage rate and material price trends, whereas contractor indexes such as the Turner index reflect low labor productivity and attendant factors.

An interesting comparison can be made of Mr. Howson's statement with that of Thurman Arnold on the cause of high construction costs in the period from 1932 to 1940. The latter states:

> "Building costs soared in 1937 and were largely responsible for reversing the upward trend of business throughout the country, not only in construction, but in all industries. A major factor in high building costs has been a series of concerted efforts to fix prices and to perpetuate antiquated methods."

---

[27] "Highlights of Construction Costs—1946," *Engineering News-Record*, April 18, 1946, p. 107.
[28] "Construction Trends," *ibid.*, August 8, 1946, p. 63.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

ENGINEERING EDUCATION

PROGRESS REPORTS OF THE SOCIETY'S PROFESSIONAL COMMITTEE ON ENGINEERING EDUCATION, FOR 1944 AND 1945

Discussion

BY WARREN RAEDER

WARREN RAEDER,[8] M. ASCE.[8a]—These progress reports deal with possibly the most fundamentally important question of policy facing engineering educators at this time—how best to prepare today's engineering students to meet tomorrow's engineering problems. It is a big problem and because of its importance the report should be studied thoroughly, particularly for implications arising from the committee's recommendations for coverage of curriculum. The committee recommends the following content for the curriculum:

Course	Percentages
Humanistic—social courses (HS)	20
Physical sciences, including geology	15
Drawing	4
Mathematics, not including trigonometry	10
Mechanics, hydraulics, and strength of materials	11
Engineering subjects, other than civil engineering	10
Civil engineering	30
	100

This discussion deals with only the first and last items. Most teachers will heartily approve of the percentages recommended for the other items. They are the minimum required for a good foundation; yet they are ample.

Consider, however, the HS value of 20%. Few engineers and fewer educators will object to the desirability of including this field of study in an engineering curriculum. Time has marched on and engineers are well established in

NOTE.—These reports were published in March, 1946, *Proceedings*. Discussion on this report has appeared in *Proceedings*, as follows: June, 1946, by Frederic Bass, and John B. Wilbur; September, 1946, by Lowell O. Stewart, and Roy M. Green; and October, 1946, by Harry Rubey.

[8] Prof., Civ. Eng., Univ. of Colorado, Boulder, Colo.

[8a] Received November 12, 1946.

a new era. Engineering has become so much a part of today's living, and it is so tied up with the growth of humanism and social development, that a groundwork of principles in the "humanistic-social" field is essential to a sound curriculum. A generation or less ago the educator left the acquiring of that field of knowledge to the enterprise of the individual. In the light of today's more complex living such a course is too haphazard. The question immediately arises, however, as to how much time can be allowed, in a four-year curriculum, to adequately handle the HS "stem"; or, conversely, shall the engineering school increase the undergraduate course of study to five years in order not to encroach unreasonably upon technical content? Assume a four-year curriculum of 17 hours per quarter in the freshman year and 18 hours for the other three years, making a total of 213 quarter hours. Deduct 6 hours for physical education, leaving 207 hours to work with. This allows 41 hours for HS subjects. One distribution might be as follows:

Course	Hours
Freshman English	9
Literature	9
Economics	9
Public speaking	3
Report writing	3
Engineering economy	3
Electives	5
	41

In looking over this table of contents one wonders about the HS attributes of some of the subjects. With literature, economics, and engineering economy there should be at least fairly general approval. Some will disagree, however, with freshman composition (probably sophomores would if consulted), public speaking, and report writing. Frankly, the writer has included these latter two subjects because, along with composition, they head the list of desirable "other subjects" advocated by engineers in answer to the questionnaire sent out by the committee and there is no place for them; that is, they do not come under the heading of drawing, mathematics, mechanics, or the other classifications. The writer thoroughly agrees with the insistence upon these subjects by those answering the questionnaire. With the electives reduced to 5 hours, however, much of the appeal of the HS group evaporates; although, of course, some may wish to substitute other subjects like history or American government for literature or economics. A year of economics, however, has been included in many curricula for many years. It seems to the writer that the HS requirements are not met nearly so well as one might wish. Certainly, to be honest with ourselves, engineering educators should question whether all the foregoing 41 hours are truly HS. Unquestionably 26 hours are; but that is $12\frac{1}{2}\%$ and not 20%.

For strictly civil engineering subjects 30% of the cirriculum is recommended; because of the hypothetical 207 hours that means 62 hours. The writer questions whether 62 hours is sufficient and submits the following

schedule of courses which might be drawn up if one is limited to that number of hours:

Subject	Hours
Surveying, plain and topographic	8
Highway engineering, and curves and earthwork	8
Structural analysis, including statically indeterminate structures	7
Timber design	4
Steel design	6
Reinforced-concrete design	8
Irrigation and drainage	2
City planning	3
Water supply and sewerage	5
Municipal and sanitary design	4
Foundations	3
Estimates and costs	2
Seminar	2
	62

Perhaps no one will agree with this schedule in its entirety; but those who may wish to cut down in some subjects, like irrigation and drainage or estimates and costs, are likely to emphasize other subjects to a greater extent or to include subjects which may not even appear in the foregoing tabulation so that the result would still be approximately the same.

A total of 62 hours is too small for comfort. For example, "surveying" has been reduced to a minimum. One answer to the surveying problem, of course, is to require attendance at a summer school of surveying. The University of Illinois at Urbana has recently started such a summer school. In other words, the curriculum is lengthened from four years to four years plus, say, one-half quarter; that is, the equivalent of 9 quarter hours, thus making 71 hours available for civil engineering subjects, or 33% of the total instead of 30%. The writer is not one of those who believes that it is sufficient to teach the principles of surveying and to "let the student get the practice after graduation." That is somewhat like teaching the principles of swimming or of public speaking without enough practice to convert the principles into good understanding and sound technique.

To indicate further how cramped a curriculum is with 30% civil engineering subjects one could comment that the foregoing schedule has eliminated such courses as building construction and bridge construction. These are descriptive courses; it is fashionable today to point out that they have little or no training value and may well be eliminated on that account. Perhaps so, and yet that information is of great assistance to a young engineer if he is not to start out in his first job at too menial a task. If he cannot run a simple survey well, with sufficient precision and without too much fussing around—if he does not know how a building is put together or what some of the regulations of the fire underwriters are—an employer may have him taking off quantities or drawing up bar diagrams longer than desirable.

The foregoing hypothetical curriculum has little hydrology except as may be included in water supply; it has no soil mechanics except as may be brought in in foundations; it has no airports except as may appear in highway engineering or city planning; nothing in evaluation; not enough time for structural analysis unless handled by the best of teachers; and possibly not enough for sanitary engineering.

It may not be necessary to include all these items in the work required for a Bachelor of Science degree. The question the writer wishes to raise is whether in a four-year curriculum an engineering school really has sufficient time to do a good job and adequately handle both the HS courses and the professional civil engineering course. Every year there are new developments in the profession. Although many of these normally belong to the field of graduate study it is not only fair but necessary that the undergraduate know about many of these developments when he receives his diploma. No longer is it possible to have him graduate without at least an introduction to such concepts as moment distribution and the Swedish slip circle. The committee found that 67 out of 114 schools required from 30% to 42.4% of the curriculum to be devoted to civil engineering subjects. In recommending that this ratio be reduced to 30%, the committee is suggesting that more than half of the 114 schools studied replace their civil engineering courses with others, presumably HS courses. The writer wishes the committee had discussed the "pros" and "cons" of a five-year curriculum although he can understand why it apparently preferred not to raise that vexatious question. It seems obvious that the committee is in favor of maintaining a four-year curriculum. It reports ("Conclusions and Recommendations"):

> "It is possible to devote approximately 20% of the time to nontechnical subjects and yet to include the fundamental technical and science content in the four-year curriculum leading to the degree of Bachelor of Civil Engineering."

To summarize, the writer wishes to raise these questions:

1. In maintaining the four-year curriculum, can engineering educators really afford to give 20% of their curriculum to HS subjects without cutting down unduly on what is after all their prime objective—to teach civil engineering?

2. Is the HS stem as presently constituted completely "humanistic-social"?

3. Are not some essential studies in the civil engineering field being omitted when this coverage is restricted to 30% of the curriculum?

4. In the light of questions 1 to 3 should not the engineering educator consider the four-year curriculum as being on probation, keeping in mind particularly the low salary status of the engineer? Is it good strategy to turn out engineers less thoroughly trained technically when engineers are already woefully underpaid? Is the engineering college making it more difficult or less difficult to raise the economic status of engineers?

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

RELIEF WELLS FOR DAMS AND LEVEES

Discussion

By John R. Charles, Horace A. Johnson, P. C. Rutledge, H. H. Roberts and Carter V. Johnson, Frank E. Fahlquist, and Harry R. Cedergren

John R. Charles,[14] Assoc. M. ASCE.[14a]—This carefully prepared report on a relatively new use for wells is commendable. Wells to supply water are as old as history; more recently wells have been designed to return water to the ground; and now wells are being designed for the relief of pressure under dams and levees.

It is interesting to note that the problems concerning determinations of permeability and other factors of water source, which increase specific capacity of the well units and provide materials to assure long life with low maintenance expense, have occurred with each of the foregoing type of well. In the modern water-supply well, the outer casings are sealed with cement grout under pressure to prevent seepage down around the casing. Bronze screens and gravel walls reduce the friction loss of entrance and increase specific capacity; outer and inner casings are constructed with welded joints throughout—all in order that the units will have the desired capacity and long life.

At the first introduction of the return well, it was thought that all these refinements might be superfluous; the first return wells were homemade affairs which quickly clogged up, and permitted the return water to escape up around the outer casings. It was soon found that construction equal to that used for the water-supply well was required to provide a constant operating return well.

Similarly, it will be found that relief wells should be constructed in a manner equal to present-day methods accepted for the construction of water-supply wells and return wells. The use of an outer casing sealed with cement grout under pressure would be superior to casing sealed with bentonite and sand mix. Bronze screens will be found necessary to assure continued life and permanent use of the relief wells; the gravel wall is generally accepted as a means of increasing specific capacity and reducing the number of wells required.

Note.—This paper by T. A. Middlebrooks and William H. Jervis was published in June, 1946, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: October, 1946, by Henry C. Barksdale, Willard J. Turnbull, and Glennon Gilboy; and December, 1946, by W. A. Wall and C. A. Stone

[14] Dist. Engr., Layne-New York Co., Inc., Pittsburgh, Pa.

[14a] Received October 25, 1946.

In 1945, three experimental wells were completed for the Corps of Engineers, U. S. Army, in the Missouri River Valley, to test the type of well best suited for the relief of pressure under levees. Relative specific capacities (in gallons per minute per foot of drawdown) were as follows:

Type	Capacity
Gravel wall (screen C)	224
"Natural gravel pack" (screen C)	68.3
Gravel wall, double cased (screen L)	360

These three wells were located 150 ft on centers in a line along the levee and were provided with 12-in. screens in each case. This test would indicate the variation in specific capacity between the gravel wall well and the so-called "natural gravel pack" as well as between various types of screens.

Relief wells should be constructed in order to provide the highest measure of efficiency in relieving water pressure. They should also be constructed so that there will be the minimum of maintenance and tested at regular intervals. As with most wells, it is to be expected that measures of standard service and cleaning will be required to keep these units in service, and ready for use. Assuredly there would be no time to clean relief wells during the flood period when the relief of pressure is most desperately needed.

Messrs. Middlebrooks and Jervis have cited the uncertainties concerning the boundaries of the seepage flow and the coefficient of permeability. Under the heading, "Design of Relief Well Systems," the authors state: "Both these considerations require the application of judgment based on experience to obtain reasonable solutions in complicated geological formations." Similar experience with water-supply wells proves the accuracy of this statement.

It is possible that some part of the pressure behind a levee may be caused by the increase of water in the natural underground formations. Heavy rainfall on the sides of the valley would result in a ground-water peak moving toward the river. The weight of the levee would help to confine this peak, and the effect of the river rising faster than the ground water would also increase the pressure immediately outside the levee. The combination of these two pressures at the line outside the levee might account, in some part, for the greater-than-anticipated pressure in this area. Similarly, the increase of natural underground water following heavy rainfall, and consequent increase of natural underground pressure, might account for the greater flows beneath dams than would be indicated by the hydrostatic pressure of the water in the reservoir.

The writer can visualize dual-purpose wells constructed to relieve pressure on dams and levees and also to supply this water, which has been filtered and purified by passage through the sands, for use in water-supply systems.

HORACE A. JOHNSON,[15] ASSOC. M. ASCE.[15a]—It has been assumed in this paper that an infinite line source is paralleled by an infinite line of wells. This

[15] Engr. (Civ.), U. S. Engr. Office, Sacramento, Calif.

[15a] Received November 1, 1946.

is a nearly true condition in many levees and simplifies the problem by making it two dimensional. However, in the design of an earth dam across a V-shaped or U-shaped valley this assumption may be considerably different from the true condition. The purpose of this discussion is to show that by use of the electrical analogy, and by constructing a three-dimensional model, the foregoing assumption may be dispensed with in certain cases, thus broadening the field of application. The authors discuss the use of the electrical analogy, but confine the discussion to a two-dimensional model. In many cases it is practically as easy to construct a three-dimensional model as it is to construct a two-dimensional model and a more exact representation and solution are obtained.

As a case in point, consider a pervious stratum with a very low dip, downstream about 2°, and striking approximately at right angles to the stream bed, which outcrops about 3,000 ft above the proposed dam. At the downstream toe of the dam in the stream bed this stratum is overlain by 100 ft of practically impervious material. With a depth of water of 125 ft in the reservoir, or a head of 225 ft to the bottom of the impervious stratum, the weight of this overlying stratum gives a factor of safety of less than 1 against uplift, assuming no loss in head through flow in the pervious stratum and no flow in the impervious stratum. Fig. 12 illustrates the conditions existing in the example. Permeability of the stratum A was determined by well tests to be 50 ft per day with a hydraulic gradient of 1. Stratum B is very impervious as attested by the fact that an artesian head of approximately 30 ft above river water level exists in stratum A. Stratum B is assumed to be absolutely impervious in the analogue.

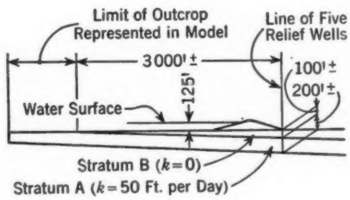


Fig. 12

With these conditions existing it is necessary to reduce the pressure beneath the stream bed and up the abutment slopes until there is sufficient material overlying stratum A to resist the uplift. Five wells spaced at 200-ft centers were tried and gave satisfactory results in pressure reduction.

An electrical analogue was set up on a scale of 1 in. equals 100 ft with 2 in. of water representing the 200 ft of stratum A. The bottom of stratum B was originally represented by the bottom of the electrical analogy tank. The model was thus built upside down, with the wells projecting up from the bottom of the electrical analogy tank. It was found more convenient to rebuild the model right side up for ease in adjusting the well penetration. The wells were represented by No. 22 gage copper wire, which is 0.025 in. in diameter. The source of inflow was represented by a copper plate 8 in. by 12 in. with 2 in. of the 12-in. dimension bent at 90°. The 8-in. by 10-in. dimension was placed at the water surface level with the 2-in. leg projecting down to the bottom of the tank. The area of this plate represented the area of the outcrop of stratum A in the reservoir.

Although the boundary conditions are not exactly represented because of the limitation of the size of the tank (40 in. by 66 in.), the effect of the imposed

boundaries can be estimated by building the model at a smaller scale. In this case another model on a scale of 1 in. equals 200 ft was constructed and tested. Results of this test indicated that the effect of the artificial boundaries

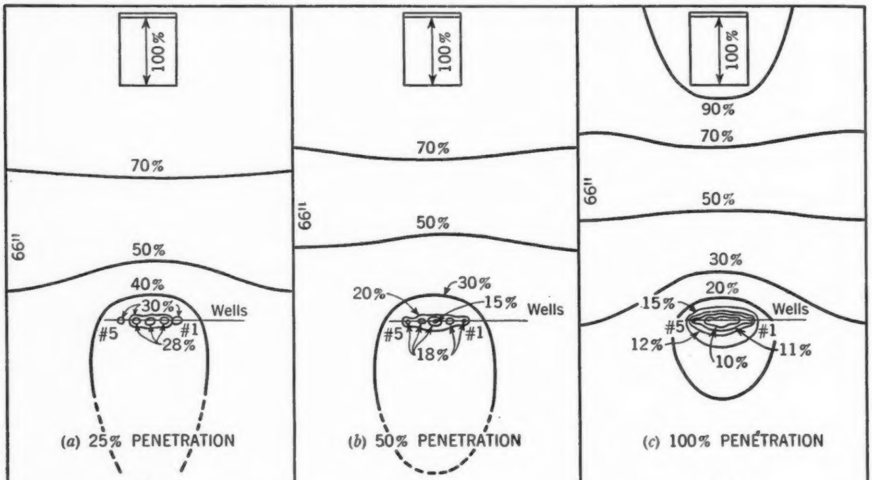


FIG. 13.—EQUIPOTENTIAL LINES FOR DIFFERENT DEGREES OF WELL PENETRATION

on the larger model (1 in. equals 100 ft) were very small.

Results of the electrical analogy tests for three different well penetrations of 50 ft, 100 ft, and 200 ft, representing 25%, 50%, and 100% penetration of the 200-ft stratum, are presented in Fig. 13. In these tests the source was at

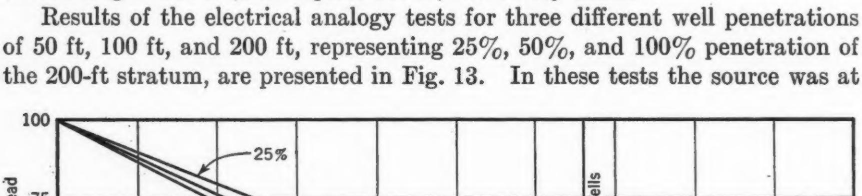


FIG. 14

100% potential and the wells at 0% potential. The other lines marked with percentage figures represent the loci of points having those percentage potentials.

In order to provide a rough comparison with Fig. 7, Fig. 14 is a plot of the potentials along an axis through the center well and the center of the source.

As would be expected, because of the finite number of wells, the "extra length" is considerably greater than that obtained by the authors, but the effect of penetration is relatively about the same.

A further difficulty with a finite number of wells is the computation of flow from each well. The computation of the total flow from all the wells can readily be made by sketching in the flow lines on the potential net. However, it is very difficult to extend this method to an individual well. The flow in individual wells, therefore, was obtained by calibrating the electrolyte and measuring the flow of current into each well. This was done by running individual wires to each well, maintaining all wells at zero potential, and reading the current for each well in turn. Total flow was also measured by measuring the current in all wells simultaneously. The results of current measurements and conversion to flow of water in Table 2 appear to be very satisfactory. The total flow was also checked by sketching the flow net.

TABLE 2.—FLOW MEASUREMENTS[a]

Well No. (Fig. 13)	(a) 25% PENETRATION; $k' V d' = 140$			(b) 50% PENETRATION; $k' V d' = 157$			(c) 100% PENETRATION; $k' V d' = 19.6$		
	I (1)	Q (2)	% (3)	I (1)	Q (2)	% (3)	I (1)	Q (2)	% (3)
1........	15.7	1.62	103.5	21.1	1.95	107.8	3.1	2.28	114.0
2........	14.9	1.54	98.5	18.6	1.71	94.5	2.5	1.84	92.0
3........	14.3	1.48	94.5	18.3	1.69	93.3	2.4	1.77	88.5
4........	14.8	1.53	98.0	18.9	1.74	96.2	2.5	1.84	92.0
5........	15.9	1.64	105.0	21.1	1.95	107.8	3.1	2.28	114.0
All.......	75.5	7.80	....	100.0	9.21	....	13.8	10.2	....

[a] $k = 50$ ft per day; $h = 125$ ft; $d = 200$ ft; $k\,h\,d = 1{,}250{,}000$ cu ft per day or 14.47 cu ft per sec; I (see Cols. 1) = measured flow of electrical current; Q (see Cols. 2) $= I \dfrac{k\,h\,d}{k'\,V\,d'}$ in cubic feet per second (see Eq. 16); and the values in Cols. 3 represent the well flow compared to the average, rated as 100.

The flow of current in the model is easily converted to water flow in the prototype since

$$\frac{Q}{I} = \frac{k\,h\,d}{k'\,V\,d'} \qquad (16)$$

in which Q is water flow in cubic feet per second; k is the effective coefficient of permeability of the pervious stratum; h is the head in feet at the source of seepage; d is the depth in feet of the pervious stratum; I is the current flow in the model; k' is the electrical conductivity of the electrolyte; V is the voltage impressed on the model; and d' is the depth of electrolyte in the model. The product $k'\,V\,d'$ can be determined directly without breaking it down into component quantities by calibrating the electrolyte in a tray of known dimensions. No absolute values of the current flow I, in amperes, were obtained in these experiments; relative values only were obtained.

Two factors proved to be troublesome in the model testing for current flow. The first of these was discovered when a calibration of the electrolyte from a cylindrical surface to a well did not check the calibration using a rectangular

tray. The apparent explanation was that the effective diameter of the model well was considerably smaller than the actual diameter of the wire representing the well. The effective diameter of the well can be computed from the formula:

$$I = \frac{2\,\pi\,k'\,d'\,V}{\log_e\left(\frac{r_c}{r_w}\right)} \qquad (17)$$

in which r_c is the inside radius of the calibrating cylinder, and r_w is the effective radius of the model well. All the factors except r_w are known or can be measured; so r_w can be determined. Eq. 17 is analogous to the formula for flow of water into a single well.[16]

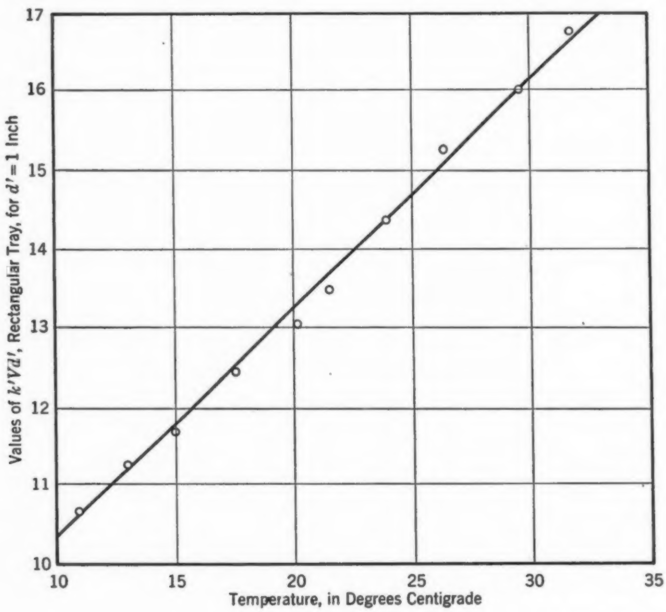


Fig. 15

A further series of tests, run to determine the effective diameter of various sizes of electrodes at several different current concentrations, showed that the effective diameter was only from 5% to 10% less than the actual diameter. Since this represents a difference of actual current flow into the electrode of only 2% or 3%, it was decided to ignore the effect of decrease in diameter as insignificant.

The second factor that must be watched in the measurement of current is the temperature of the electrolyte. It was found that a very few degrees of difference in temperature had an appreciable effect on the current flow. It is

[16] "Methods for Determining Permeability of Water-Bearing Materials," *Water-Supply Paper No. 887*, U.S.G.S., U. S. Govt. Printing Office, Washington, D. C., 1942, p. 79.

necessary, therefore, to calibrate the electrolyte at the temperature at which the experiment is run. Fig. 15 shows the effect of temperature on the conductivity of the electrolyte used which was tap water of unknown analysis.

P. C. Rutledge,[17] Assoc. M. ASCE.[17a]—An outstanding feature of this paper is a prudent use of rather complex theoretical analyses. The authors have emphasized the uncertainties in the field conditions and the necessity for tempering with judgment a design based on their analyses. They claim as a significant advantage the flexibility of relief wells because modifications of an installation can easily be made to meet conditions disclosed by field use for the control of underseepage.

One question which the writer pondered after studying this excellent paper was: "On what can one base one's judgment of a relief well design?" Engineering judgment should be based on accumulated experience and field observations, or on an orderly method of estimating the effects of the several variables involved. Both bases for judgment seem to be lacking for this problem. In particular, the uncertainties in the character and the extent of the pervious soil layer drained by relief wells have effects that cannot be evaluated directly by judgment. Briefly, these uncertainties about the actual pervious soil layer are five:

(*a*) Degree and effects of stratification;
(*b*) Effects of lenticular deposits of silt and clay;
(*c*) Effective depth of the entire layer;
(*d*) Effective distance from the source to the wells; and
(*e*) Effective permeability of the layer.

It may even be argued that these uncertainties invalidate the analyses made by the authors. The writer does not agree with such an argument, however, and will endeavor to show in this discussion that a rational basis for judgment can be formulated and that the effects of the uncertainties previously listed are not serious.

Neglecting for the moment the uncertainties in the pervious soil layer and assuming, in its place, a homogeneous layer of measurable dimensions, the variables that affect the performance of drainage wells are:

1. Length of the pervious layer, s, in feet;
2. Depth of the pervious layer, d, in feet;
3. Effective coefficient of permeability of the pervious layer, k, in feet per minute;
4. Spacing of the wells, a, in feet;
5. Percentage penetration of the wells into the pervious layer;
6. Difference in head between the source and the discharge point, Δh, in feet;
7. Effective radius of the well, r_w, in feet; and
8. Frictional head loss in the well screen and riser pipe.

[17] Prof., Civ. Eng., The Technological Inst., Northwestern Univ., Evanston, Ill.

[17a] Received November 4, 1946.

The inflow into a well is determined by the first seven variables when the difference in head is that between the source and the point of inflow. The effective radius of the well is rarely the actual pipe radius. It may be larger if the well pipe is surrounded by a filter or smaller for a screened well in contact with the soil. The radius of the well pipe governs frictional head losses for flow up the pipe but otherwise has nothing to do with the well inflow.

Eight variables, all of which definitely affect performance, are too many to think about as clearly and concisely as one must in the exercise of judgment. To simplify the problem the authors have introduced two very useful terms:

(1) "Carrying capacity" or maximum possible discharge of the pervious layer under any given conditions; and
(2) "Extra length" which relates well discharge to the "carrying capacity."

The writer will use these two terms to simplify the problem to a greater extent. The authors have shown that the carrying capacity, Q_M, and the discharge of one well in a line of wells, Q_a, both in cubic feet per minute, can be expressed as follows:

$$Q_M = \frac{k \, \Delta h \, a \, d}{s} \qquad (18a)$$

and

$$Q_a = \frac{k \, \Delta h \, a \, d}{s + \text{extra length}} \qquad (18b)$$

Further simplification is obtained if Eqs. 18 are written in terms of

$$\mathbf{W} = \frac{Q}{k \, \Delta h \, d} \qquad (19)$$

in which **W** is a dimensionless number which might be called the "well production number." It can readily be shown that the solution of every problem of steady flow into a well can be expressed in terms of **W** and that, for every problem, **W** depends only on geometrical considerations. Thus, the production number for the "carrying capacity" is

$$\mathbf{W}_M = \frac{a}{s} \qquad (20a)$$

For a single well completely penetrating a homogeneous pervious layer fed by an infinite line source, the production number is

$$\mathbf{W}_1 = \frac{2\,\pi}{\log_e\left(\frac{2\,s}{r_w}\right)} \qquad (20b)$$

For a single well on the perpendicular bisector of a line source of finite length, *a*, the production number is

$$\mathbf{W}_{1a} = \frac{2\,\pi}{\log_e\left[\frac{2\,s}{r_w}\left(1 + \frac{4\,s^2}{a^2}\right)\right]} \qquad (20c)$$

For one well in a line of wells spaced at a distance, a, on centers and completely penetrating a pervious layer fed by an infinite line source, the production number is

$$\mathbf{W}_a = \frac{a}{s + \frac{a}{2\,\pi} \log_e \frac{a}{2\,\pi\,r_w}} = \frac{a}{s + \text{extra length}} \quad \text{(21}a\text{)}$$

From Eq. 21a it is obvious that the extra length for fully penetrating wells is

$$\text{Extra length} = \frac{a}{2\,\pi} \log_e \frac{a}{2\,\pi\,r_w} \quad \text{(21}b\text{)}$$

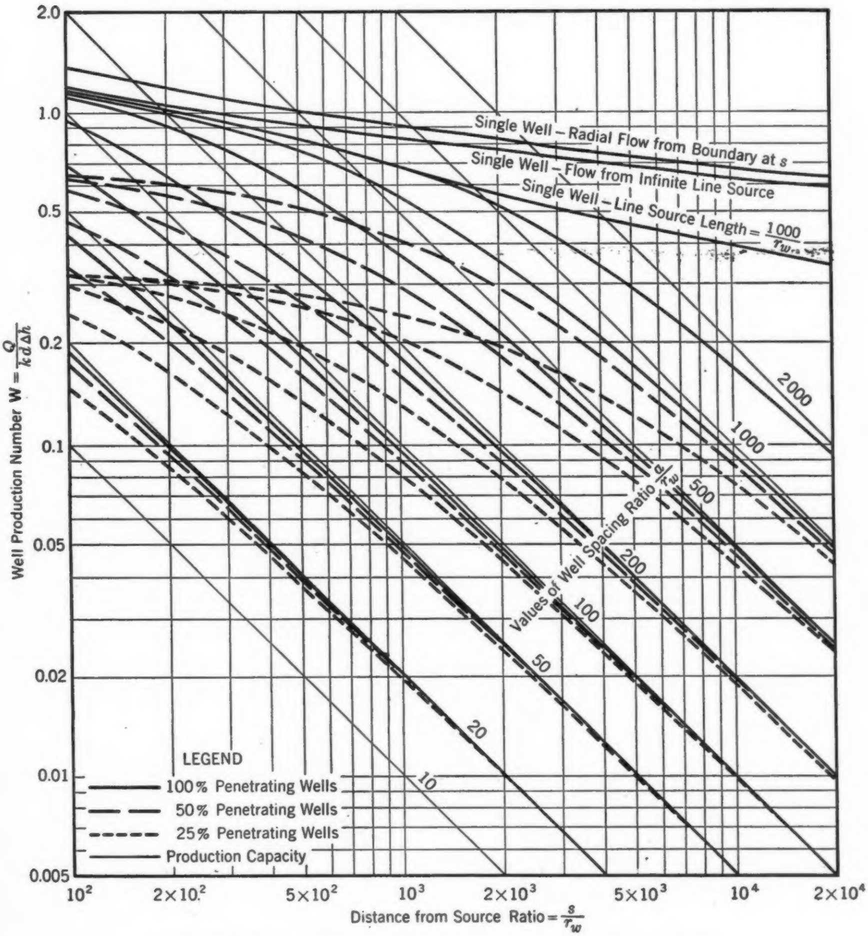


FIG. 16.—WELL PRODUCTION NUMBERS; FOR STEADY FLOW ONLY

The basic solutions for the foregoing cases, and for many others, have been presented elsewhere[18] by M. Muskat. Fig. 16 shows values of the well produc-

[18] "The Flow of Homogeneous Fluids Through Porous Media," by Morris Muskat, McGraw-Hill Book Co., Inc., New York, N. Y., 1st Ed., 1937, pp. 178, 191, and 529.

tion numbers for various values of s and a expressed as ratios of the effective well diameter. This chart shows the dimensional conditions under which fully penetrating wells in a line produce practically the equivalent of the carrying capacity of the pervious layer. As the well spacing is increased, the productions of individual wells in a line approach, but do not equal, the production of a single well fed by an infinite line source or by a line source with a length of the order of 1,000 ft. The production numbers of single wells are indicative of performance but are not numerically significant. The reason is that the solu-ions for single wells assume that the well has been flowing long enough to have affected the heads in the entire pervious layer. Even with very pervious materials, this condition is not reached within any practical period of time.

The disadvantages of Fig. 16 are that a comparison of well production to the carrying capacity is more significant than the actual well production numbers and that, as the authors have stated, wells do not fully penetrate the pervious layer in most field installations. For fully penetrating wells the ratio of well production to carrying capacity is

$$\frac{\mathbf{W}\,a}{\mathbf{W}_M} = \frac{Q\,a}{Q_M} = \frac{\dfrac{s}{r_w}}{\dfrac{s}{r_w} + \dfrac{a}{2\,\pi\,r_w}\log_e \dfrac{a}{2\,\pi\,r_w}} \qquad (22a)$$

This ratio might be called the "relative production." For partly penetrating wells the relative production can be computed from the experimental curves in Fig. 8:

$$\frac{\mathbf{W}_{ap}}{\mathbf{W}_M} = \frac{Q_{ap}}{Q_M} = \frac{\dfrac{s}{r_w}}{\dfrac{s}{r_w} + \dfrac{a}{r_w} \times \dfrac{\text{extra length}}{a}} \qquad (22b)$$

in which the extra lengths are taken from the experimental curves for various values of $\frac{a}{r_w}$ and of percentage penetration. An analysis of fully and partly penetrating wells, made by replotting the experimental data in Fig. 8, indicates that, within the limits of accuracy required by any practical applications, well spacing and degree of penetration are directly compensating variables in the relative production. Thus relative production depends on a term of the form:

$$\frac{a}{r_w}\,\frac{100}{\%\ \text{penetration}} = \text{inflow length} \qquad (23)$$

In other words, the inflow lengths of well systems with 100% penetrating wells 80 ft on centers, with 50% penetrating wells 40 ft on centers, or with 25% penetrating wells 20 ft on centers are identical. Fig. 17 is a plot of relative production in terms of s/r_w and inflow lengths. This one chart summarizes completely the performance of wells penetrating homogeneous pervious layers of measurable dimensions.

Uses of Fig. 17.—The purpose of relief wells is to reduce to safe values the hydrostatic pressures in underlying pervious soil layers on the land side of

levees and the downstream side of dams. A continuous vertical discharge face which completely intersects the pervious layer, the condition assumed for the carrying capacity, limits the head at and on the landward side of the discharge face to the discharge head. The carrying capacity and the discharge of a line

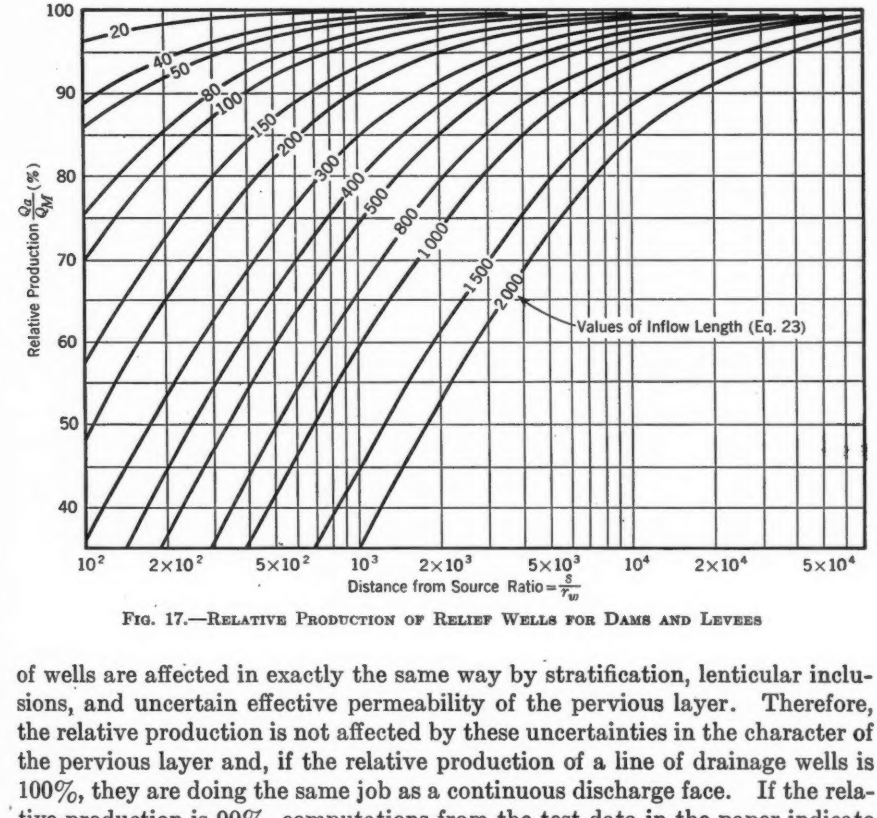


FIG. 17.—RELATIVE PRODUCTION OF RELIEF WELLS FOR DAMS AND LEVEES

of wells are affected in exactly the same way by stratification, lenticular inclusions, and uncertain effective permeability of the pervious layer. Therefore, the relative production is not affected by these uncertainties in the character of the pervious layer and, if the relative production of a line of drainage wells is 100%, they are doing the same job as a continuous discharge face. If the relative production is 90%, computations from the test data in the paper indicate that midway between wells the maximum head in excess of the discharge head will not exceed 20% of the total change in head for any combination of well spacing and percentage penetration. Since the test data presented do not include penetrations less than 25%, this and all other conclusions in regard to effects of partial penetration may not be valid for penetrations less than this value.

Assume that s is 1,200 ft and that wells with an effective radius of 0.25 ft will be used. Taking a relative production of 90% as a design value, the inflow length from Fig. 17 is 800. Since the inflow length is expressed in terms of effective well radius, the ratio of well spacing to degree of penetration is 200. Therefore, 100% penetrating wells spaced 200 ft on centers or 50% penetrating wells spaced 100 ft on centers, or 25% penetrating wells spaced 50 ft on centers could be used. From Fig. 8 the percentage of excess head midway between

wells for these three possible designs will be 13.5%, 10.5%, and 8.3% of the total head change, respectively. Next assume that 50% penetrating wells spaced 100 ft on centers are selected for the design but that the depth of the pervious layer actually is twice that determined from field explorations and used in design. The result will be that relative production is 81% instead of 90% and the excess head is 19% instead of 10.5% of the total change in head. Similarly, if the actual effective length of the pervious layer were only 600 ft instead of 1,200 ft for the same design, the relative production would be reduced from 90% to 82% and the excess head would again be increased to 19% of the total change. In the rather remote case that 100% errors on the unsafe side are made in both the depth and the effective length of the pervious layer, the relative production would be reduced to 67% and the excess head would be 31% of the total change.

It should be apparent from this numerical example that extreme errors in determinations of the dimensions of the pervious layer do not impair seriously the effectiveness of a design based on the erroneous dimensions. This fact, coupled with the fact that use of the relative production automatically eliminates the effects of stratification, lenticular inclusions, and uncertain effective permeability of the pervious layer, should answer any objections to relief well designs based on the uncertainties of the pervious layer. In addition, it has been shown by A. F. Samsioe[19] and A. Casagrande[20] that the effects of stratification can be compensated for and the problem reduced to one of flow through a homogeneous, isotropic material, by reducing true dimensions parallel to the stratification to effective dimensions by a factor $\sqrt{\frac{k\ (\text{min})}{k\ (\text{max})}}$. Therefore, the effect of stratification on partly penetrating wells cannot be greater than that of a change in effective length of the previous layer, by the ratio $\sqrt{\frac{k\ (\text{min})}{k\ (\text{max})}}$.

One word of caution is necessary, however. These conclusions are valid for the primary purpose of relief wells, which is to eliminate dangerous hydrostatic pressures due to underseepage. They do not apply to the actual discharge quantities from the wells. The discharge quantities will vary directly with every uncertainty in the pervious layer. Therefore, variations of 1,000% from the anticipated discharge quantities should not be considered unusual. Since the discharge quantity governs the frictional head loss in upward flow through the well riser pipe, a liberal factor of safety should be used in selecting the size of the well pipe.

Summary.—In conclusion, the writer believes that the authors have made a significant contribution in their development of the use of drainage wells for the relief of dangerous pressures due to underseepage. Their conclusions are valid and they have justly advocated the use of drainage wells only for the specific problem of relieving pressures in underlying pervious layers. Drainage wells usually will have little or no effect on seepage which actually passes through a dam or levee. However, for the specific problem to which drainage

[19] "Einfluss von Rohrbrunnen auf die Bewegung des Grundwassers," by A. F. Samsioe, *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 11, No. 2, 1931, pp. 124–135.

[20] "Seepage Through Dams," by A. Casagrande, *Journal*, New England Water Works Assn., Vol. 51, No. 2, June, 1937, pp. 151–154.

wells apply, the authors have been modest in their claims and, if anything, have underestimated the flexibility of the system.

H. H. ROBERTS,[21] ESQ., and CARTER V. JOHNSON,[22] ESQ.[22a]—The pressure-relief well system at the Fort Peck Dam is cited by the authors as being the most outstanding example of the use of such wells. A brief statement regarding the Fort Peck installation and its effectiveness is also given in the paper under discussion.

Information that has been obtained during the period that the relief wells have been in operation at the Fort Peck Dam verifies the effectiveness of this method of relieving excessive subsurface pressure in the critical area immediately downstream from a large dam. Where data from field investigations for a proposed dam indicate the possibility of underseepage, under similar conditions, consideration should be given in the design stage to the immediate installation of a few pressure-relief wells for the purpose of obtaining accurate information for the design and installation of a complete relief-well system. The information obtainable from the initial wells taken during the period that the reservoir is being filled, could be used for the final design based on the criteria and methods given in the paper. This procedure should reduce to a considerable extent the need for extensive underground explorations and laboratory tests of the materials that would be required if the permanent system were to be designed in its final form along with the plans for the dam.

As a matter of interest the equations and charts given in the paper have been applied to the well system at the Fort Peck Dam, using the heads and gradients that existed on July 29, 1946 (highest elevation of water in the Fort Peck Reservoir to that date, 2,232.25 ft), the relief well system discharge, and available data regarding the pervious strata. Correlation of the computed characteristics of the system with the actual was very good.

Data collected since the installation of the pressure-relief wells show that the total discharge of the well system is proportional to the reservoir elevation; it has also been observed that the water elevation in the river downstream from the dam has a material effect on the well discharge. Piezometer pipes both upstream and downstream from the line of wells furnish evidence of the change in well discharge due to fluctuating reservoir and tailwater elevations. Piezometer pipes 50 ft upstream from the line of wells are used to measure the average hydrostatic pressure in the well area; and since the installation of the well system this hydrostatic pressure has never indicated a head in excess of 7 ft above the average ground surface. Maximum and minimum discharges of 12.35 cu ft per sec and 8.5 cu ft per sec, respectively, have been observed for a reservoir fluctuation of 62 ft in which time the reservoir reached an elevation only 18 ft below the maximum level. It has been estimated that the well system discharges from 75% to 85% of the total amount of seepage water that approaches the system. The remainder of the seepage water continues downstream, confined within the pervious strata by the thick overlying layer of

[21] Lt. Col., U. S. Corps of Engrs., Fort Peck Dist., Fort Peck, Mont.

[22] Engr. (Soil Mechanics), U. S. Corps of Engrs., Fort Peck Dist., Fort Peck, Mont.

[22a] Received November 6, 1946.

clay. Consequently, an average hydrostatic pressure can be allowed downstream from the dam which will produce a head that extends above the ground surface without creating a wet and boggy condition.

The present well system at Fort Peck is in excess of the capacity actually needed to relieve the pressure to the required level. This allows a factor of safety for well failures which have occurred due to deterioration of the screen and casing. Several wells have failed because of such deterioration since the original installation; and additional wells have been installed with a more permanent type of screen and casing to assure adequate capacity should too many of the original wells fail. When an individual well fails, the total discharge of the system does not change appreciably and the load is absorbed by all the wells in inverse proportion to their distance from the failed well. When a new well is added to the system the flow in the other wells is reduced in the same manner. Individual wells vary considerably in discharge. The factors that have the greatest effect on the discharge are the effective head producing discharge (which can be varied for individual wells by changing the elevation of flow line), the length of screen, and the type of material which the individual well taps. Since the well system is adequate and the discharge is produced by a very small head, the wells are sensitive to small changes.

In the design of a well system certain fundamental factors enter into the problem; and a thorough understanding of these is necessary in order that a reasonable design may be developed. Furthermore, the effectiveness of the equations and charts for an actual design would depend, of course, on the accuracy of the basic data. The equations and charts in the paper can be used to illustrate the effect of these factors on a system of wells by assuming different conditions and solving for the result. As previously stated, when checking the equations and charts with observed data at Fort Peck, the results checked very well with the actual. This check indicates that, if the fundamental data and tests are reliable, an adequate system of relief wells can be designed by the methods outlined. However, wherever practicable, it is considered desirable actually to install a minimum number of wells and to utilize data from their operation during the initial filling of the reservoir to complete the final design.

FRANK E. FAHLQUIST,[23] M. ASCE.[23a]—The design of underground drainage facilities for partial yet adequate control of underseepage pressures is presented by the authors. The paper summarizes the very valuable data of experimentation and experience that was accumulated for several years by the United States Engineer Department. Although the authors have presented the important features of relief well design, it is believed they could also contribute valuable additional data and discussion concerning the fabrication and installation of wells. In particular, comments on the several available types of well intakes, such as metal screens, concrete pipe with porous wall, and perforated tile would be appreciated. Comments as to the preparation

[23] Cons. Engr. and Geologist, Riverside, R. I.

[23a] Received November 13, 1946.

and the stabilization of ground surrounding the well intakes, either by development methods or by placement of graded filter materials, would be in order.

The authors' explanation of Fig. 1 can be further illustrated by the application of two formulas showing the relations of permeability in stratified materials. In general, the permeability in an unstratified or homogeneous deposit is the same in all directions. Geologically, there can be exceptions to this rule but they are not important. If, in an otherwise homogeneous deposit, stratifications of less permeability occur, the permeability in a direction that is across the bedding is much less than that in a direction parallel to the bedding, as shown by the two equations,[24]

$$k_H = \frac{1}{H}(k_1 d_1 + k_2 d_2 + k_3 d_3 + \cdots + k_n d_n) \ldots\ldots\ldots (24a)$$

and

$$k_V = \frac{H}{\frac{d_1}{k_1} + \frac{d_2}{k_2} + \frac{d_3}{k_3} + \cdots + \frac{d_n}{k_n}} \ldots\ldots\ldots\ldots\ldots (24b)$$

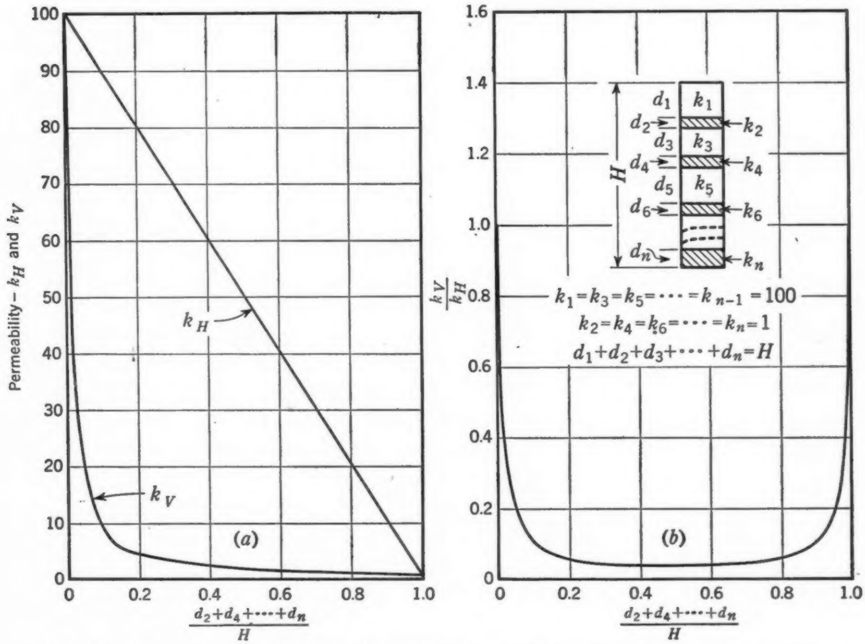


FIG. 18.—HORIZONTAL AND VERTICAL PERMEABILITY IN A PERMEABLE SEDIMENT CONTAINING THIN STRATIFICATIONS OF RELATIVELY IMPERMEABLE MATERIAL

in which k_H is the permeability parallel to bedding or horizontal direction; k_V is the permeability perpendicular to bedding or vertical direction; k_1, k_2, k_3, $\cdots$ k_n represent the permeability of individual layers; d_1, d_2, d_3, $\cdots$ d_n represent the thickness of individual layers; and $H = d_1 + d_2 + d_3 + \cdots + d_n$ is the total thickness.

[24] "Theoretical Soil Mechanics," by Karl Terzaghi, John Wiley & Sons, Inc., New York, N. Y., 1943, pp. 243–244.

The relations expressed in Eqs. 24 are shown graphically in Fig. 18. In Fig. 18 the aggregate thickness of relatively impermeable stratifications is

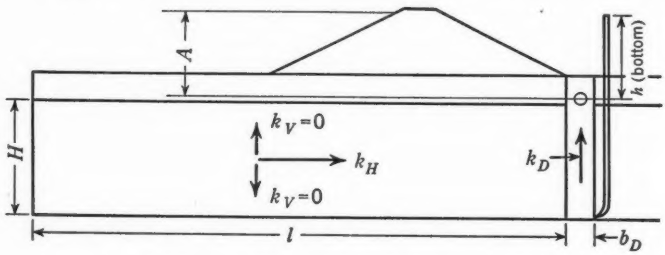


FIG. 19.—FACTORS INVOLVED IN THE DERIVATION OF EQS. 25

indicated by $d_2 + d_4 + \cdots + d_n$, and the fractional part of these thicknesses to the total thickness is $\frac{d_2 + d_4 + \cdots + d_n}{H}$. If the permeability of the pervious beds is assumed to be alike (for demonstration purposes) and 100 times that of the impervious stratifications, also assumed to be alike, the impermeable occurrences of only a fractional part of the total thickness will cause a very appreciable drop in the k_V-value. In the k_H-value there is only a very small reduction, whereas a large reduction is noted in the ratio, $\frac{k_V}{k_H}$. It does not matter whether the impermeable component occurs as a single thick unit, or in numerous thin stratifications. It is this phenomenon, as indicated by the authors, that produces the inadequate vertical drainage which causes flotation of the overburden with consequent piping and formation of boils. In constructing relief wells, the impermeable stratifications or layers are pierced, and facilities are provided for the escape of ground water that is confined in the permeable beds.

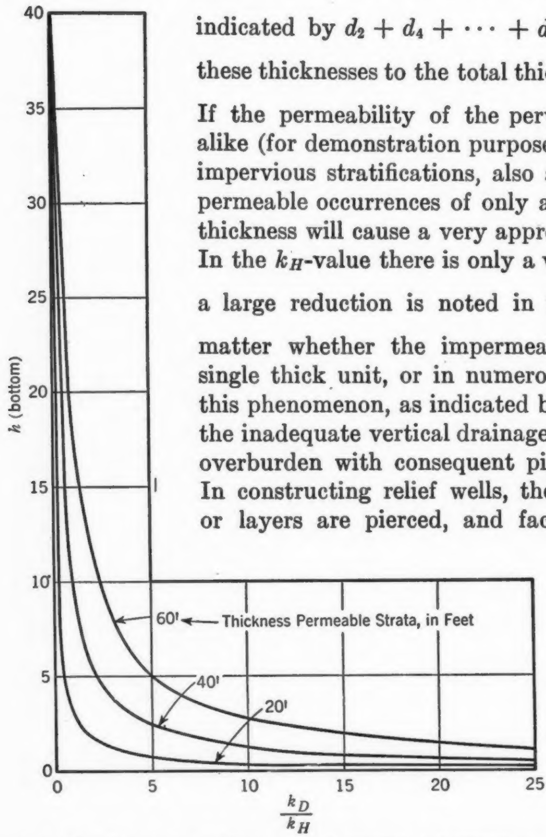


FIG. 20.—RELATION OF PIEZOMETRIC PRESSURE, *h* (BOTTOM) AT UNDERSIDE OF PERMEABLE STRATA TO REQUIRED RATIO OF PERMEABILITIES OF DRAINAGE ZONE AND FOUNDATION

The average over-all permeability of a bedded sediment is related to the degree of stratification; the amount of fine particles mixed with coarser particles, such as sand and gravel; and the density or porosity of the various beds. The last two factors can be demonstrated easily in the laboratory, and the effect of the first factor has been discussed. Considering these factors and also the manner in which sediments, either fluvial or glacial in origin, are frequently found

in nature, the writer has investigated the theoretical factors involved, and the construction methods required, for creating a continuous and deep drainage zone, in the underground, for the interception of seepage.

The method would involve systematic jetting in a vertical zone, generally less than 10 ft wide, in which, by controlling the jetting process, stratifications could be disrupted, fines floated and washed away, and the entire zone rendered more porous than the adjacent foundation materials. It would not be applicable to all materials or conditions—for example, it would not be practicable to attempt such relief in strata in which sand was predominant. However, there are situations where, because of geologic processes, the deposits are of stratified sand and gravel with varying proportions of fines (silt and clay), in dispersed and concentrated occurrence. For conditions such as these, the relation of the several factors, shown in Fig. 19, can be expressed as:

$$\frac{h\ (\text{bottom})}{A} = 1 - \left(\frac{2\, e^{-\beta H}}{1 + e^{-2\beta H}}\right) \qquad (25a)$$

$$i_0 = \beta\, H \left(\frac{1 - e^{-2\beta H}}{1 + e^{-2\beta H}}\right) \qquad (25b)$$

and

$$\beta = \sqrt{\frac{k_H}{k_D}\, \frac{1}{l\, b_D}} \qquad (25c)$$

in which h (bottom) is the piezometric pressure at the bottom or underside of the pervious strata; i_0 is the hydraulic gradient at the elevation of the drain pipe or ditch; A is the head acting; H is the thickness of the pervious strata; e is the base of Naperian logarithms = 2.71828; k_H is the horizontal permeability of the pervious strata (vertical permeability assumed to be zero); k_D is the permeability of the drainage zone; b_D is the width of the drainage zone; β is a number defined by Eq. 25c; and l is the length of the seepage path. For a convenient solution of Eqs. 25, curves of $\beta\, H$ versus h (bottom) A, and $\beta\, H$ versus $i_0\, H/A$ should be prepared.

To consider a hypothetical case, assume $A = 40$ ft; $l = 300$ ft; and $b_D = 8$ ft. The relation between the piezometric pressure, h (bottom), at the underside of the permeable strata to the ratio, k_D/k_H, is demonstrated by the curves in Fig. 20. A change in permeability of the pervious foundation strata, within the narrow drainage zone, from $k_D = 1$ to $k_D = 4$, produces a very appreciable decrease in the piezometric pressure acting at the bottom or underside of the pervious strata. The practicability of producing such a change in permeability, of course, is the important uncertainty. To the writer's knowledge, such a construction has never been attempted and consequently there is not an experience record by which to judge its feasibility; but by reasoning alone it would appear that, in certain types of material (as previously discussed), a moderate change from $k_D = 1$ to $k_D = 3$ could be produced. The jetting operation would be accomplished by from four to six jets, held rigidly at the same spacing, with provisions for the flexible control of pressure and flow, and operated simultaneously.

The curves in Fig. 21 can be used for determining the width of drainage zone, b_D, required for a length, l, from 100 ft to 1,000 ft. The ratio of permeability of drainage zone to pervious foundation strata is assumed to be unity. If a greater change in permeability is believed possible, the width of zone required is less—for example, if the permeability of the zone is doubled, the width may be reduced by one half.

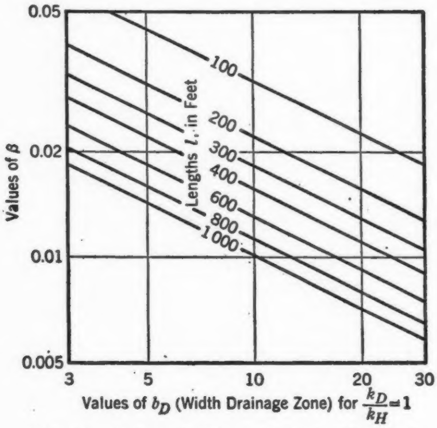


FIG. 21.—REQUIRED WIDTH OF DRAINAGE ZONE

As a further illustration, consider a typical problem of an earth dam, for which $A = 100$ ft; $H = 40$ ft; and $l = 800$ ft. At the downstream toe there is a trench 8 ft deep, and the drainage zone is constructed in such a manner that the h (bottom) at the underside of the pervious stratum is 8 ft. Therefore, $h\,(\text{bottom})/A = 8/100 = 0.08$. From Eqs. 25, or design curves based on them, $\beta\, H = 0.41$ and $\beta = \beta\, H/H = 0.41/40 = 0.01$. By reference to Fig. 21, for $\beta = 0.01$ and $l = 800$ ft: $b_D = 12$ ft when $k_D/k_H = 1.0$, and 6 ft when $k_D/k_H = 2.0$.

Considering both the theoretical and practical aspects of the continuous and deep drainage zone, it is obvious that:

(*a*) All seepage may be intercepted;
(*b*) The zone can be varied in width to take care of varying amounts of seepage;
(*c*) The zone is formed in place by modifying the foundation materials; and
(*d*) The drainage zone can be easily incorporated in other drainage control facilities.

The several uncertainties and disadvantages are as follows:

1. Feasibility of construction depends on the character of foundation materials—the most suitable primary materials being sand-gravel mixtures having dispersed silt and clay, or silt and clay interstratifications;

2. The change of permeability in a drainage zone, relative to adjacent foundations, would be difficult to determine, but the problem is not insuperable; and

3. The cost may be greater, and yet may be less, than that for relief well installations, depending on the number of wells required, and the necessity for placing filters, or stabilizing the ground around well intakes.

Acknowledgment.—The differential equation leading to Eqs. 25 was developed and solved by R. A. Barron, Jun. ASCE.

HARRY R. CEDERGREN,[25] Assoc. M. ASCE.[25a]—The subject of relief wells, in this paper, is presented in a straightforward, yet reserved, manner. The statement (under the heading, "Design of Relief Well Systems") that "* * * the well system is very flexible and more and larger wells can be added at any time if the first installation proves inadequate" is one of the keys to the potential value of relief well systems.

During 1946 the writer was engaged in design studies for a 25-mile system of levees along the Columbia River in Washington. Explorations of proposed sites for levees to protect agricultural and industrial areas revealed extremely heterogeneous deposits of gravelly materials underlying a relatively impervious blanket of fine-grained soils. The overlying blanket varied in thickness from less than 1 ft in isolated areas to 15 ft or 20 ft in others, and averaged about 8 ft. The gravelly formations ranged from well-graded, relatively impervious, silty sandy gravel to extremely open-graded strata of 0.5 in. (+) to 4-in. (+) material with only a trace of fines. The open-graded strata generally did not exceed 1 ft or 2 ft in thickness, and occurred in lenses or pockets of limited extent. The individual lenses or pockets were more or less interconnected, and permitted a relatively free passage of water in the horizontal direction. Underlying the gravelly formations at depths of from 30 ft to 60 ft below the ground surface were beds of clay that were indicated by incomplete explorations to be impervious and continuous.

Complicating the problem, much of the area to be protected was under irrigation, which produced a substantial flow of water toward the river. This flow added to potential construction problems and necessitated even greater seepage protection than would otherwise have been necessary. Where the impervious bed was from 30 ft to 40 ft deep, the use of an impervious cutoff, in combination with shallow drains for the removal of runoff, seemed the most promising solution. Blanketing the riverbank and beaches with relatively impervious soil was considered as the primary control in some areas. In other places, the use of deep relief wells as the principal controls appeared to be the most economical and adequate solution. One of the principal advantages of relief wells on this project is their flexibility. Prediction of the probable seepage quantities, even with the aid of considerable explorations and extensive field and laboratory permeability tests, could be nothing more than a reasonably good guess. Consequently, it was essential that the drainage, or relief system, be of such type that it could be "tailored" to fit the needs of the job as those needs were finally determined by observation of the prototype.

A second advantage of relief wells on this project is their effectiveness in relieving hydrostatic pressures in highly stratified deposits. Shallow drains in such formations are relatively ineffective, since even a thin impervious stratum separating the bottom of a drain from pervious underlying strata can seriously reduce the efficiency of the drain. However, wells drilled into the pervious strata provide a direct means of escape for seepage and can reduce uplift pressures in the foundation substantially.

[25] Cons. Engr., O. J. Porter & Co., Sacramento, Calif.

[25a] Received December 2, 1946.

Should an initial relief well installation not be adequate to control seepage during maximum stages of river or reservoir, this inadequacy frequently can be predicted by piezometric observations during low and intermediate stages. Such a possibility was demonstrated by the authors under the heading, "Examples of the Use of Relief Wells."

Relief wells should be constructed with considerable care in accordance with specifications prepared by competent soil engineers. When used on important structures, in which their continued functioning over a long period is required, each well should be accessible for cleaning and repairing. If wells are installed beneath an embankment section, vertical riser pipes should extend above the embankment to provide access to the wells. With this provision, and with adequate attention to their design and construction, relief wells can be used with confidence in the most important structure. Periodic observation of seepage pressures within and beneath important earth dams and levees, and frequent observation of the effluent of relief wells and drains, should be routine matters for attendants.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

RIGID-FRAME STRUCTURES SUBJECT TO NONUNIFORM THERMAL ACTION

Discussion

By A. L. Miller

A. L. Miller,[7] M. ASCE.[7a]—A distinct service has been rendered by the author in directing attention to problems arising from differential temperature change, and by presenting a feasible interpretation and theoretical evaluation. Necessarily, theory is abstract. Limited by hypothesis and assumption, it produces results which must be interpreted with due regard for these limitations. In addition to the generally recognized stipulations of the theory of flexure and continuity, the materials are presumed to possess their usual properties and behavior above the ordinary range of temperature. Also, it will be observed that the dimensional changes of the convex face of a member are assigned to its centroidal axis. Both premises are justifiable and undoubtedly give results that are more severe than the real effects. In other words, the theoretical results predict the upper limit of effects within which to exercise engineering judgment.

Subscribing to the proposals of the paper, this discussion aims to validate the results of the illustrative examples and to indicate procedures for the application of "elastic line" methods of calculation for those who prefer them to the "elastic energy" method used by the author. "Moment Distribution"[8] and "The Theorem of Joint Translation"[9] (an extended form of the theorem of three moments) are convenient tools for this purpose. The examples of the paper will be solved by moment distribution, a most useful elastic line method. Solution by the theorem of joint translation will not be illustrated but the essential load term will be derived for use by those who are familiar with the theorem.

An initially straight member, when subjected to differential thermal action throughout its length, tends to assume a circular arc unless resistance is afforded

Note.—This paper by Carl C. H. Tommerup was published in June, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1946, by I. Oesterblom, and Frank R. Higley.

[7] Prof. of Mechanics and Structures, Dept. of Civ. Eng., Univ. of Washington, Seattle, Wash.

[7a] Received October 25, 1946.

[8] "Moment Distribution," *Bulletin No. 64*, Eng. Experiment Station, Univ. of Washington, Seattle.

[9] "The Theorem of Joint Translation," *Bulletin No. 89*, Eng. Experiment Station, Univ. of Washington, Seattle.

by its attachments or supports. Since the curvature is relatively small, the theory applicable to straight members applies with negligible error. The extent to which the member is prevented from assuming the span-end slopes of the circular arc determines the span-end moments induced by the action and the consequent stresses and deflections throughout a rigid frame.

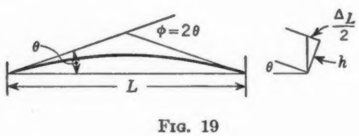


FIG. 19

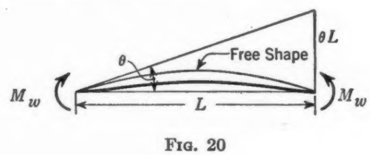


FIG. 20

Following the notation of the paper, the geometry of an initially straight member of uniform section which assumes a circular arc without stress when subjected to thermal differential is expressed (see Fig. 19):

$$\Delta_L = L\,\alpha\,\Delta t; \qquad \theta = \frac{\Delta_L}{2\,h} = \frac{L\,\alpha\,\Delta t}{2\,h}; \qquad \text{and } \phi = 2\,\theta = \frac{L\,\alpha\,\Delta t}{h} \ldots (22)$$

in which θ and ϕ are expressed as slopes, consistent with the basic procedures of the theory of flexure. Accordingly, the fixed-beam moment $M_{F,N}$ caused by the thermal differential, for use in the moment-distribution method, is as follows (Fig. 20): By the moment-area principle,

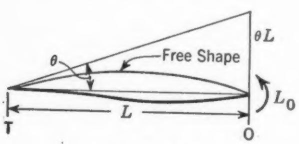


FIG. 21

$$\theta\,L = \frac{M_w}{E\,I}\,\frac{L^2}{2}; \qquad \text{and } M_w = \frac{2\,E\,I\,\theta}{L} \ldots (23a)$$

Therefore—

$$M_w = \frac{E\,I\,\alpha\,\Delta t}{h} \ldots\ldots\ldots (23b)$$

—with rotational signs determined by inspection. Eq. 23*b* demonstrates that the fixed-beam moment M_w caused by the thermal differential is no function of the span length.

The load term, L_o, for use with the theorems of two moments, three moments, and joint translation is derived as follows (Fig. 21): By the moment-area principle,

$$\theta\,L = \frac{L_o}{E\,I}\left(\frac{L}{2}\right)\frac{L}{3}; \qquad \text{and } L_o = \frac{6\,E\,I\,\theta}{L} \ldots\ldots\ldots\ldots (24a)$$

Therefore—

$$L_o = \frac{3\,E\,I\,\alpha\,\Delta t}{h} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (24b)$$

The differential temperature Δt that causes upward deflection is positive. The rectangular frame of Fig. 1 is solved by moment distribution as follows:

The joint coefficients of the span ends are computed in the conventional manner, being 0.429 and 0.571 for the 160-in. and 120-in. members, respectively. The carry-over factors are 0.5 throughout. The fixed-beam moments are

found by the equation,

$$M_w = \frac{E\,I\,\Delta t}{h} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (25)$$

Thus, $M_w = \dfrac{2{,}000 \times 14{,}230 \times 0.0000079 \times 400}{20} = 4{,}496.68$ kip-in. for all members since each is subjected to the same thermal differential and possesses the same values of E, I, and h. Since no balancing moment appears in the first operation, subsequent distributions and carry-overs are zero, giving 4,496.68 kip-in. as the precise result at each span end. When arithmetical and geometrical approximations are eliminated, the solutions presented in the paper produce this result.

The rectangular frame with tie rod and hinges (Fig. 9) is analyzed by moment distribution as follows:

The joint coefficients of each span end are computed in the usual manner with due regard for the pins at points B and E. Carry-over factors are all 0.5

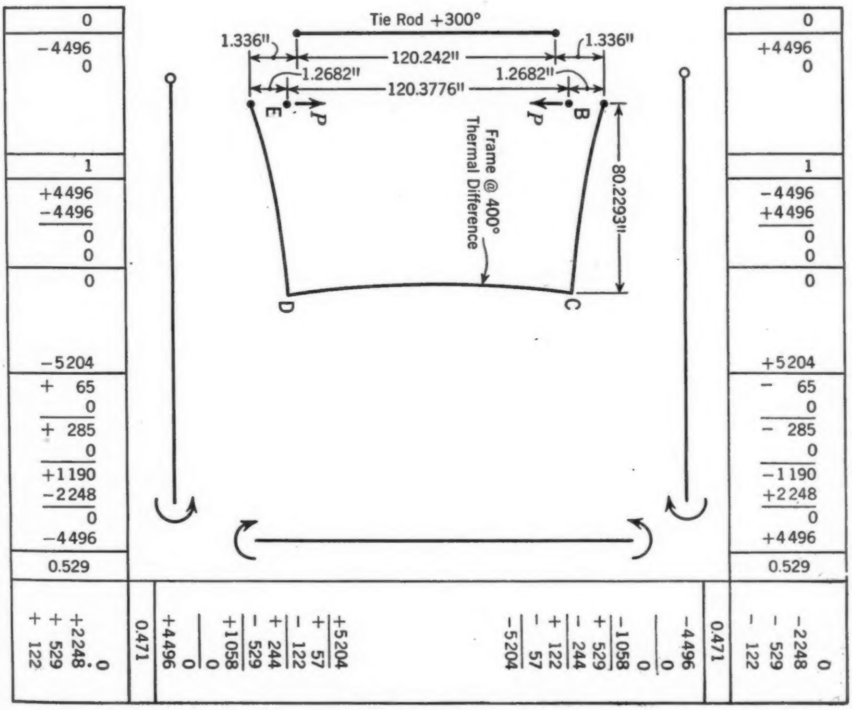


FIG. 22

except those to the pins, B and E. The form of tabular procedure shown in Fig. 22 is convenient for any rigid frame and is self-explanatory to one familiar with the method of moment distribution. The boxes representing the joints provide a space in which to record the balancing moments; the joint coefficients

appear on the span ends adjacent to the joints; and the steps in the successive cycles proceed outward from the joints in the sequence: Fixed-beam moment—distribution—carry-over—distribution—carry-over—distribution—etc. The span-end moment is the sum of the items at that span end. The number of cycles is the number of balancing moments appearing in the joint. Each step must proceed concurrently in all members throughout the structure.

The fixed-beam moments are calculated by Eq. 25, which yields 4,496 kip-in. for members BC and ED and 5,204 kip-in. in member CD. Rotational signs are determined by inspection. The computation is completed as follows:

Item	Description	
1	The load P (in kips), necessary to cause a displacement Δ of 1.2682 in., is $\frac{5{,}204}{80.2293}$, or	65.00
	Displacement Indexes, in Inches per Kip—	
2	Bending throughout the frame is $\frac{1.2682}{65.0}$, or	0.0195
3	Compression in member CB is $\frac{P\,L}{2\,A\,E} = \frac{1 \times 60.189}{1{,}060 \times 2{,}000}$, or	0.000028
4	Bending and compression combined	0.019528
5	Elongation of tie rod is $\frac{P\,L}{2\,A\,E} = \frac{1 \times 60.121}{5.5 \times 30{,}000}$, or	0.000364
6	Total index (items 4 and 5)	0.019892
7	The displacement Δ (in inches) of the frame plus the tie rod is $T_{BE} \times 0.019892$, or	1.336
8	The tie-rod force T_{BE} (in kips) is $\frac{1.336}{0.019892}$, or	67.0
9	The bending moment M, in kip-inches, is	5,360
10	If the tie rod is lengthened 1.20 in., item 5 becomes $\frac{1 \times 60.721}{5.5 \times 30{,}000}$, or	0.000368
11	The tie-rod force (item 8) becomes $\frac{1.336 - 0.60}{0.019896}$, or	37.0
12	The corresponding bending moment is	2,960

The rectangular frame with two cross walls in Fig. 13 must be analyzed in two separate solutions. The span-end moments due to curvature, computed in Fig. 23(a), are superimposed on the moments due to the differential expansion between the cross walls and end walls in Fig. 23(b) to produce the final results in Fig. 24. The magnitude of differential displacement between the ends of the 229.5-in. members is 0.015 in., based on the dimensions shown in Fig. 15. Because the frame is biaxially symmetrical, only one quarter, as shown in Fig. 17(b), will be needed for the solutions since a carry-over can be regarded as reflected back from an axis of symmetry with reversal of sign.

The joint coefficients are found in the usual manner and all carry-over factors are 0.5. Fixed-beam moments due to curvature are found by Eq. 25 for each member, and three cycles of operation follow.

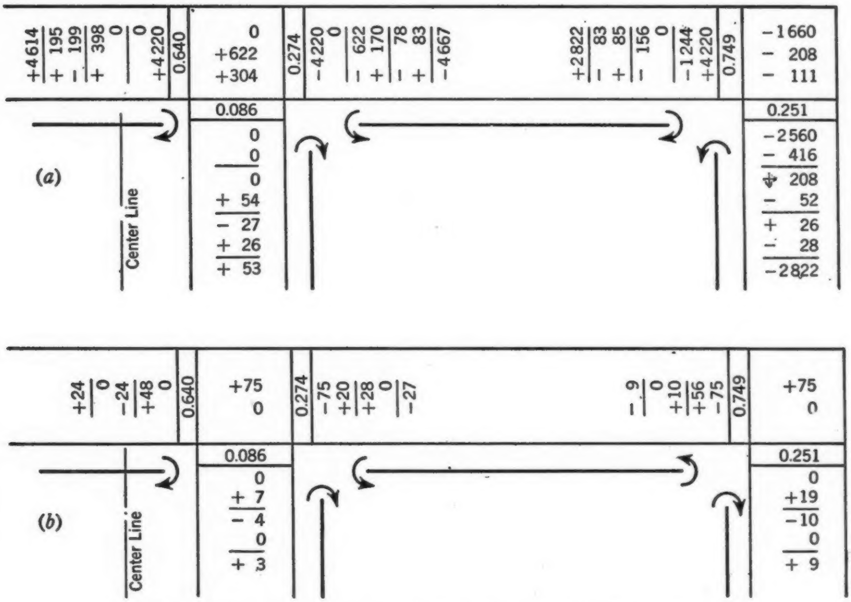


FIG. 23.—RECTANGULAR FRAME WITH TWO CROSS WALLS; CHECK SOLUTION

The coefficients of the frame having been found, the fixed-beam moments due to end translation are computed by:

$$M_d = \frac{6\,E\,I\,\Delta}{L^2} \qquad (26)$$

and found to be 75 kip-in. for each of the 229.5-in. members when = 0.015 in. Δ The term is zero for all other members. Two cycles produce precise results.

Combining the results of Figs. 23(*a*) and 23(*b*) and expressing them in accordance with the general sign convention for comparison with Fig. 17(*b*) produces the moments shown in Fig. 24.

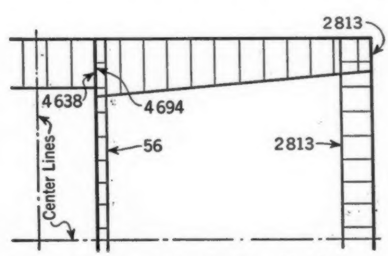


FIG. 24.—FINAL MOMENTS IN KIP-INCHES

The results appearing in this discussion differ slightly from those of the paper. However, it will be observed that the problems are very sensitive numerically, slight initial variations producing noticeable final differences. Absolutely precise numerical results were obtained by means of the theorem of joint translation (not illustrated herein). The precise value of the moment for

the rectangular frame with the tie rod in a rectangular position is 5,188.48 kip-in. which produces a value of T_{BE} = 66.8 kips and M = 5,344 kip-in.

Precise results for the span-end moments with two cross walls are: M_{BA} = 4,687.12; M_{AB} = 2,810.64; M_{BH} = 57.61; and M_{BE} = 4,636.6—all in kip-inches.

Elastic line methods have the distinct advantage of eliminating inaccuracies inherent in the complicated geometrical calculations required by elastic energy methods. Reliability of theoretical results is enhanced and the required time and effort greatly reduced thereby.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

DESIGN OF PLYWOOD I-BEAMS

Discussion

BY SIDNEY NOVICK

SIDNEY NOVICK,[15] JUN. ASCE.[15a]—The method for determining stiffener spacing (see heading, "Web Stiffeners") is based on the theoretical buckling behavior of plywood plates in shear. Fig. 4 appears to have been derived by finding simultaneous values of $\frac{d_w}{s}$ and $\frac{d_w}{b_w}$ for a fixed value of $v_c = 5 \times 240$, rather than "* * * by computing values of v_c for various combinations of $\frac{d_w}{b_w}$ and $\frac{d_w}{s}$," as stated by the author (paragraph following Eqs. 9). These curves, therefore, give spacings such that the theoretical factor of safety against shear buckling will be at least 5 when the horizontal shear is equal to one fifth of the maximum horizontal shear stress of the material, v_h (max)—that is, when $v_h = \frac{v_h\,(\text{max})}{5} = 240$ lb per sq in. The value of Fig. 4 is limited by its theoretical basis and by the failure to provide for increasing spacings when the design value v_h is less than $\frac{v_h\,(\text{max})}{5}$.

In 1944, the Army-Navy Civil Committee (ANC) on Aircraft Design Criteria, summarizing a more comprehensive study by the Forest Products Laboratory,[16] presented an "Experimental Buckling Curve"[17] which demonstrates that plywood beam webs cannot be assumed to buckle always at stresses close to the values predicted by theory. Actual shear stresses at buckling are equal to, or greater than, the predicted values only when the plate dimensions are in such relation as to cause a limiting value to be exceeded; below this limit, the actual stresses are less than predicted.

NOTE.—This paper by Howard J. Hansen was published in June, 1946, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1946, by Dick W. Ebeling, I. Oesterblom, and C. J. Hogue.

[15] Structural Engr., General Panel Corp., New York, N. Y.

[15a] Received November 1, 1946.

[16] "Design of Plywood Webs in Box Beams," *Mimeograph No. 1318* (and supplements), Forest Products Laboratory, Madison, Wis., 1943–1944.

[17] "Design of Wood Aircraft Structures," *Bulletin No. 18*, Army-Navy-Civil Committee on Aircraft Design Criteria, Washington, D. C., 1944, Fig. 2–41.

In the notation of the paper, the limit above which the theoretical buckling values are valid is that $\frac{d_w}{d_{w_o}}$ is equal to 2.2, in which d_{w_o} is the theoretical depth between flanges at which the critical buckling stress v_c would be equal to the shear strength of the material, v_h (max); that is,

$$d_{w_o} = \left(\frac{4\, u_\theta \sqrt{m_1\, m^3_2}}{b_w\, v_h\, (\text{max})} \right)^{\frac{1}{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

Fig. 5 has been plotted from the ANC curves[17] to show the variation of the ratio $\frac{v_c\,(\text{actual})}{v_c\,(\text{theoretical})}$ (denoted by r_v) with $\frac{d_w}{d_{w_o}}$.

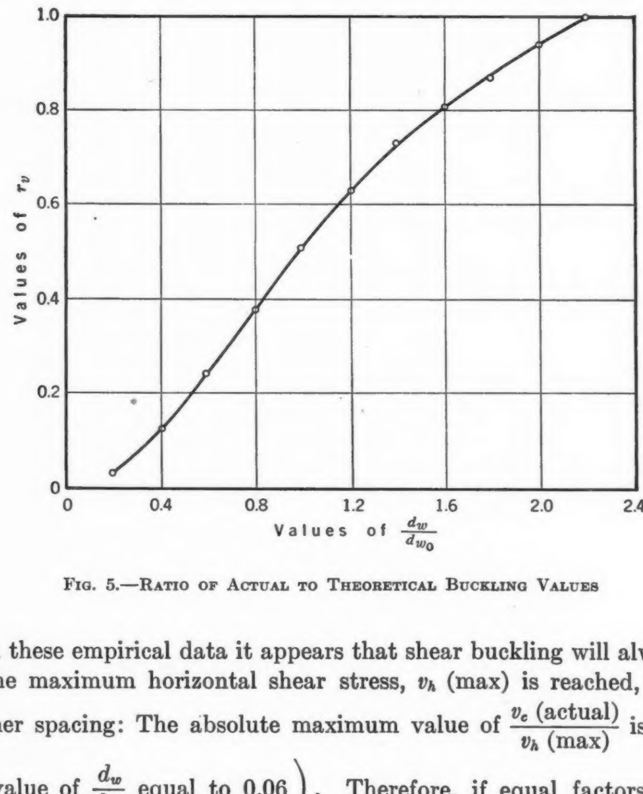


FIG. 5.—RATIO OF ACTUAL TO THEORETICAL BUCKLING VALUES

From these empirical data it appears that shear buckling will always occur before the maximum horizontal shear stress, v_h (max) is reached, regardless of stiffener spacing: The absolute maximum value of $\frac{v_c\,(\text{actual})}{v_h\,(\text{max})}$ is only 0.83 $\left(\text{for a value of } \frac{d_w}{d_{w_o}} \text{ equal to } 0.06\right)$. Therefore, if equal factors of safety against horizontal shear and shear buckling are desired, the value of the design horizontal shear, v_h, must be taken into account in determining stiffener spacing, and the maximum value of v_h must always be limited to less than $0.83 \times \frac{v_h\,(\text{max})}{5}$.

With these considerations in mind, Fig. 6 has been prepared to supplant Fig. 4. The same types of plywood and design constants as used by the author were employed; they will enable the designer to determine the spacing of stiffeners after v_h and the ratio $\frac{v_h}{v_h\,(\max)/5}$ $(= p_v)$ have been found. For commercial grades of plywood, v_h (max)/5 should be taken as the maximum safe horizontal shear stress recommended for that particular grade.

The curves in Fig. 6 were obtained as follows: From the theoretical shear buckling equation, the buckling stress varies inversely as the square of the depth between flanges; therefore,

$$\frac{v_c\,(\text{theoretical})}{v_h\,(\max)} = \left(\frac{d_{w_o}}{d_w}\right)^2 \ldots\ldots(15a)$$

If the value of v_h is such that $\frac{v_h}{v_c\,(\text{actual})/5} = 1$, then

$$p_v = \frac{v_h}{v_h\,(\max)/5} = \frac{v_c\,(\text{actual})/5}{v_h\,(\max)/5} = \frac{v_c\,(\text{actual})}{v_h\,(\max)} \ldots\ldots\ldots(15b)$$

Since $\frac{v_c\,(\text{actual})}{v_c\,(\text{theoretical})} = r_v$,

$$p_v = r_v \times \frac{v_c\,(\text{theoretical})}{v_h\,(\max)} = \frac{r_v}{\left(\frac{d_w}{d_{w_o}}\right)^2} \ldots(16)$$

For each type of plywood and for various ratios of $\frac{d_w}{s}$, values of $\frac{d_{w_o}}{b_w}$ were computed by a method recommended by the Forest Products Laboratory[17,18] and

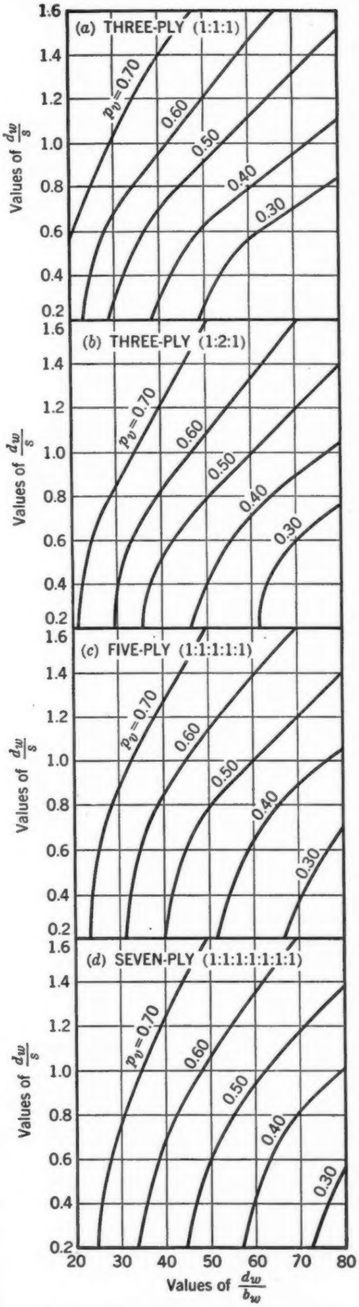


FIG. 6.—SPACING OF STIFFENERS IN PLYWOOD BEAMS

[18] "Buckling of Flat Plywood Plates in Compression, Shear, or Combined Compression and Shear," *Mimeograph No. 1316* (and supplements), Forest Products Laboratory, Madison Wis., 1942–1945.

equivalent to that used by the author. These values checked the $\frac{d_w}{b_w}$-values in Fig. 4 by a small margin. Then, for various ratios of $\frac{d_w}{b_w}$, values of $\frac{d_w}{d_{w_o}}$, r_v, and p_v were found, successively. By interpolation between the values of p_v for each combination of $\frac{d_w}{b_w}$ and $\frac{d_w}{s}$, the curves for even-numbered values of p_v, as given in Fig. 6, were found.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

SPACE RESECTION PROBLEMS IN PHOTOGRAMMETRY

Discussion

BY RALPH O. ANDERSON

RALPH O. ANDERSON,[6] Assoc. M. ASCE.[6a]—The noteworthy fact about the work reported by Professor Underwood is the solution of the space equations without resorting to partial differential calculus (Eq. 22). The implications of the foregoing statement are of technical importance in more ways than one. The advantages thus gained are as follows: High tilt in high relief may be computed with the same amount of work as that required for low tilt in low relief; and the method is considerably more simple in both derivation and application than the calculus method.[7] The first innovation consists of developing the ground pyramid for the purpose of determining the lengths of the three pyramid edges (LA, LB, and LC, Fig. 1). The suggested method will yield the correct lengths when the calculus method of adjustment is applied.

However, the graphical method will yield fairly close results. It was found that with careful drafting these pyramid edge lengths could be determined graphically within 10 ft. The errors were found to be 5 ft, − 4 ft, and 10 ft for the three edges. For example, lay off the sloping ground lengths on three transparent strips to some convenient scale, such as 1 in. equals 1,000 ft. Starting at a random point on leg A determine length A′ by fitting the sloping ground lengths between their respective pyramid edges. Repeat the operation using the starting length (leg A) equal to $\frac{1}{2}$ (LA + LA′), and repeat until LA equals LA′. This repetition is quite simple and can be done rapidly. Errors as great as 10 ft should not cause more than a 3-min tilt error. This is quite permissible in photogrammetry.

It was also found unnecessary to assume that k vanishes to zero in Eq. 22. Starting with a graphically determined exposure station expressed as a function of the principal point (letting k equal zero): $\Delta X = 252.22$, $\Delta Y = 262.20$ and $\Delta Z = -6.52$. The resulting exposure station coordinates are: $X_L = 34{,}463.22$,

NOTE.—This paper by P. H. Underwood was published in September, 1946, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1946, by Earl Church.

[6] Mathematician II, TVA, Chattanooga, Tenn.

[6a] Received November 2, 1946.

[7] "Manual of Photogrammetry," Am. Soc. of Photogrammetry, Chapter XII, 1944, p. 536.

Y_L = 11,245.20, and Z_L = 15,424.48. Repeating the solution and letting k = 66,964: X_L = 34,463.20, Y_L = 11,245.25, and Z_L = 15,419.74. These values agree precisely with the values computed by means of Professor Church's method.[7]

Professor Underwood's method of determining the tilt, when the exposure station coordinates are known, is excellent; but his method of adjustment is not clear to the writer. By referring to Figs. 5 and 12, it should be obvious that angles m_a and M_A do not lie on the same plane. Therefore, the incremental difference cannot be plotted on line ao extended (Fig. 12) without introducing an error. However, there is nothing else to do but plot the angles on their respective principal point radials. Consequently the three raised perpendiculars will not intersect at a common point. Therefore, it is necessary to apply an adjustment. Using the approximate v-position (Fig. 12) as a point within the triangle of error, compute a tilt and swing. Locate the nadir point v (f tan t from o) on the photograph (contact size). Drop perpendiculars from v to the three radials (oa, ob, and oc). Measure the lengths of these dropped perpendiculars (w) and also the lengths from the foot of these dropped perpendiculars to their respective control points (a, b, and c) and denote them as l. The angle M_A, when projected to oa, then becomes:

$$M'_A = M_A \left(\frac{l}{\frac{w^2}{2\,l} + l} \right) \qquad (34)$$

For radial ao: M_A = 1,281.1; l = 3.2 in.; and w = 0.18 in. Therefore,

$$M'_A = 1{,}281.1 \left(\frac{3.2}{3.20506} \right) = 1{,}279.08 \text{ min of arc}$$

$$m_a = 1{,}243.22$$

$$m_a - M'_A = -\ 35.86 \text{ min of arc.}$$

Using the revised value (M'_A, M'_B, and M'_C), the raised perpendiculars intersect to form a very small triangle of less than 0.01 in. in depth. All other data being correct, this would mean a tilt error of only 0° 00′ 15″, which is negligible. In large nadir throws or large tilts, possibly a second or third repetition would be needed to make the triangle of error vanish.

It is hoped Professor Underwood will treat this matter in his closing discussion.

Determination of Tilt.—For purposes of comparison a numerical example of computations to determine tilt by the "dropped perpendicular" method[8] is presented in Table 6.

Values under "First Determination" pertain to the initial computation using the principal point as the argument. Under "Second Determination," point v and the tilt i are used as arguments. Datum ratios (Table 6(a)) in the first determination are obtained by dividing the datum scales S_d by S_{do}. The value of S_{do}(1,723 ±) is determined by interpolating (visually) the datum scales

[8] "Applied Photogrammetry," by Ralph O. Anderson, Edward Bros., Inc., Ann Arbor, Mich.

(situated at their respective scale-point positions) into the principal point. The ratios of the second determination are determined as a function of the three computed tilt-axis scales (S_{di}), one for each check line. The computed tilt-axis scale for a given line is:

$$S_{di} = \frac{S_d}{1 \pm M_1 \tau} \quad \text{(35)}$$

in which M_1 is the normal distance from the tilt axis to its corresponding scale

TABLE 6.—Dropped Perpendicular Method of Computing Tilt; Numerical Example

(a) Datum Ratios—R_d

Symbol[a]	Line ab		Line bc		Line ca	
	First determination	Second determination	First determination	Second determination	First determination	Second determination
P	7.6640		5.3695		6.2150	
D	13,137.22		9,137.46		10,609.77	
S_p	1,714.147		1,701.734		1,707.123	
h_e	129.393	128.821	173.962	174.663	97.606	94.672
h_e/f	15.684	15.615	21.086	21.171	11.831	11.475
δ_d	1,729.831	1,729.762	1,722.820	1,722.905	1,718.954	1,718.598
R_d	1.003965	1.003465	0.999896	0.999487	0.997652	0.996989
M_1	......	1.145	......	−0.200	......	−0.965
$1 + M_1 \tau$	......	1.00331	......	0.999422	......	0.997209
S_{di}	......	1,724.054	......	1,723.905	......	1,723.408
Average S_{di}	$S_{do} = 1723\pm$	1,723.789				

(b) Tilt Determination

Symbol[a]	First Determination		Second Determination	
	$R_H - R_{int}$	$R_H - R_L$	$R_H - R_{int}$	$R_H - R_L$
	1.003965	1.003965	1.003465	1.003465
	0.999896	0.997652	0.999487	0.996989
Difference	0.004069	0.006313	0.003978	0.006476
l'	3.315		3.312	
C_1 (in.)	0.004069 (3.315)/0.006313 = 2.137		0.003978 (3.312)/0.006476 = 2.034	
l (in.)	2.183		2.140	
Sin t	0.006313 (8.25) /2.183 = 0.02386		0.006476 (8.25) /2.140 = 0.02497	
t	1° 22′		1° 26′	
s	225° 00′		225° 40′	

[a] In addition to the notation of the paper: P is the photographic length of the check line; D is the horizontal ground distance; $S_p = D/P$; t is the tilt angle; S_{di} is the tilt axis scale; R_H, R_{int}, and R_L represent the highest, intermediate, and lowest datum ratios, respectively; l' represents the length between the scale points conforming to R_L and R_H; and l represents l' projected normal to the line of constant ratios. The lengths l' and l are determined graphically when the datum scales are computed.

point and τ equals the sine of the tilt angle divided by f. The mean of the three S_{di}-values (1,723.789) is used as S_{di}. Accordingly, for a given check line,

$$R_d = \frac{S_d}{S_{di}} \text{ (average)} \quad \text{(36)}$$

The flying height above sea level is equal to H = (average) $S_{di}(f)$ + datum elevation = 15,421.3 ft. This is in error 1.6 ft.

The results of the first determination would generally be accepted in most photogrammetric work. The tilt and swing error is only 0° 05′ and 1° 00′, respectively. However, the second determination, which can be computed in a few minutes, will yield more accurate results. The resulting errors of tilt and swing (of the second determination) are 0° 01′ and 0° 40′, respectively. In addition, a more accurate determination is possible by applying the corrections, ΔM, ΔM_c, and h_c. These corrections are not needed in the current example because of the low tilt and relief.

When horizontal and vertical control is to be extended (without the aid of ground surveys except at flight ends) by means of computed tilts, elevations, and flying heights, all refinements must be used. A method of control extension (horizontal and vertical) is possible, using a triple parallax differential equation in conjunction with various expedients. This method is exact mathematically, but lends itself to a rapid graphical determination. The graphic errors introduced are well within the allowable tolerance. These tests were made on hypothetical examples. The photographic lengths were used to the nearest thousandth of an inch (computed) but all work from there on was performed graphically wherever possible. In a stereoscopic model of 6.5 sq miles, it was found that the maximum error in horizontal or vertical position was about 3 ft. When all values are computed to seven decimal places, the resultant position errors are less than a thousandth of a foot. The latter is really not an error but a difference due to fact that figures beyond the seventh decimal places were ignored. The method is then numerically proved.

The triple parallax differential equation consists of conjugate differences of three values which are readily determined graphically. Some simple arithmetical computations are also needed in the course of computing the elevation difference between two image points.

Professor Underwood's paper is most welcome, as diversified methods of spatial mathematics of the photogrammetric problem are practically nonexistent. The only precedent is the solution conceived by Professor Church, cited by the author.[7] Therefore, a comparison of Professor Underwood's method (A) against Professor Church's method (B) is unavoidable.

The early objective of method (A) is to determine the correct lengths of the three ground pyramid edges. These lengths can be determined graphically with a surprising degree of accuracy and also by means of a calculus adjustment. The graphically determined lengths will be close enough for all practical purposes. Knowing the correct (or nearly correct) pyramid edges, Eqs. 22 (method A) are solvable without excessive work. This solution is a near approach to a direct solution even though it consists of two parts—(1) letting k equal zero; and (2) inserting the value of k. When the value of k is inserted, the resolution is quite simple as only the constant terms are refactored.

The spatial equations in method (B) are mathematically correct but the complexity forbids a direct solution. Therefore, the partial differential calculus method of adjustment is employed. The approximate exposure station coordinates are obtained by means of the well-known, graphical three-point method. These coordinates conform approximately to the principal point ground values.

The exposure station coordinates, as initially used, are then in error. These errors, which are solved for as differentials, vary as the magnitude of the tilt. By definition, a differential is the smallest assignable value of the variable approaching zero as a limit. Therefore, differential equations express precise conditions only when the variable approaches zero as a limit. As these differentials grow in magnitude, the accuracy of the differential condition diminishes. By this reasoning the number of repetitions required by method (B) varies as the magnitude of tilt. As before stated, the equations of method (A) are in error initially as k is dropped. However, when it is inserted the initial error vanishes at a rapid rate. Re-solution is quite simple as only the constant terms are affected and they alone are factored by the same initial factors. It may then be said that method (A) is more efficient than method (B) in solving for the exposure station. However, it must be remembered that method (A) utilizes the lengths of the three pyramid edges. Therefore, in order to compare the two methods it is necessary to contrast the efficiency (ease of computation) of determining the lengths of the pyramid edges (A) against the two different exposure station computations. Method (A) will probably be more efficient than method (B). The latter would be definitely so if the pyramid edges were determined graphically and if a tolerance of from 3 min to 5 min were allowed on the tilt. The graphical method of determining tilt, method (A), is decidedly more efficient than method (B). The tilt (using exact pyramid lengths) could be determined graphically within $0° 0' 30''$ by method (A).

The general trend in photogrammetric analysis leans toward the promotion of streamlined procedures in the general direction of extending horizontal and vertical control without the aid of ground surveys other than at the flight ends. Much has been done in this direction.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

EXPRESS HIGHWAY PLANNING IN METROPOLITAN AREAS

Discussion

BY RALPH R. LEFFLER

RALPH R. LEFFLER,[26] ESQ.[26a]—Pavement-minded engineers, paving bureaus, and paving and earth-moving interests have derived much profit and power from the decentralizing of the big city. These interests have almost complete control (officially and otherwise) of planning for recentralizing (rehabilitating) the big city for the "general welfare of all." It is not surprising that puny planning, thin thinking, and verbose vaporings characterize the offerings of such of these interests as are typically selfish and bureaucratic. Mr. Barnett's paper makes determined and fairly successful efforts to avoid the selfish and bureaucratic.

Nevertheless, Mr. Barnett's paper is unintentionally quite "pavement minded." It lacks sufficient "structural mindedness"—giving scant consideration to the merits of elevated highways (on columns) and none at all to the overwhelming advantages of two-deck elevated highways (on columns) in big cities. Elevated highways, particularly two-deck elevated highways, are the product of structurally-minded as distinguished from pavement-minded engineers.

The paper treats "Parking in the Central Business District" quite cautiously, briefly, and very inconclusively under the heading, "Terminal Facilities." Extended exploration of the merits of elevated highways, especially of two-deck elevated highways (on columns), would probably have led Mr. Barnett to much more conclusive and structurally-minded ideas on "Parking in the Central Business District" on the second, third, fourth, and fifth floors of existing and future buildings and on the "setback" roof of future buildings.

The features and merits of two-deck elevated highways (on columns) have been discussed by the writer elsewhere.[27,28] Thorough study will convince almost anyone, except a paving bureaucrat, that the structurally-minded engineer can be very helpful in recentralizing the decadent big city—if given the opportunity.

NOTE.—This paper by Joseph Barnett was published in March, 1946, *Proceedings*. Discussion on this paper has appeared as follows: May, 1946, by Harry W. Lochner, and Fred Lavis; September, 1946, by Homer M. Hadley, Donald M. Baker, W. J. Van London, Merrill D. Knight, Jr., and R. H. Baldock; and November, 1946, by Jacob Feld, Harold M. Lewis, Theodore T. McCrosky, Spencer A. Snook, Lawrence S. Waterbury, Bernard L. Weiner. and George H. Herrold.

[26] Engr. of Structural Design, The San. Dist. of Chicago, Chicago, Ill.

[26a] Received November 8, 1946.

[27] "Superhighways in Widely Built-Up Areas," by Ralph R. Leffler, *Bulletin No. 74*, Am. Road Builders Assn., 1941, p. 5.

[28] "Two-Deck Elevated Highways," by Ralph R. Leffler, *Illinois Automobile Club Magazine*, Spring, 1945, p. 3.